CHAPTER 20

Volume and Surface Area of Solid Figures

Solid

Anything that occupies space is called a **Solid**. In addition to area, a solid figure has volume also. It has three dimensions namely, length, breadth and height. For solid two different types of areas namely, lateral surface area or curved surface area and total surface area are defined.

1. Prism

A solid having two congruent and parallel faces, called bases and whose other faces, the lateral faces are parallelograms, formed by joining corresponding vertices of the bases is called a **Prism**.

2. Right Prism

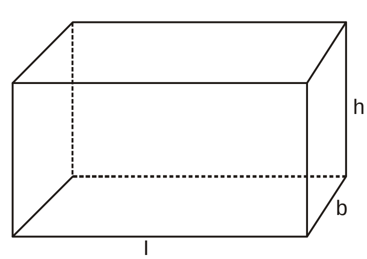
A prism in which bases are perpendicular to the lateral edges is called a **Right Prism**. The base of the prism can be a polygon.

In a right prism

(i) Number of lateral surfaces = Number of sides of the base of the prism
(ii) Total number of surfaces of a prism = Number of lateral surfaces + 2
(iii) Lateral surface area = Perimeter of base × Height
(iv) Total surface area = Lateral surface area + 2 (Area of base)
(v) Volume = Area of base × Height

3. Cuboid

A right prism in which the base is a rectangle is called a **Cuboid**. If l is the length and b the breadth of the base and h the height, then



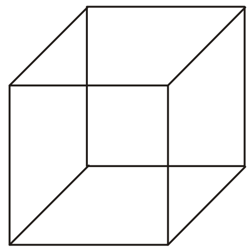
Lateral surface area = $2(l + b)h$ sq unit

Total surface area = $2(l + b)h + 2lb = 2(lb + bh + lh)$ sq unit

Volume = lbh cu unit

The longest diagonal of the cuboid = $\sqrt{l^2 + b^2 + h^2}$ unit

4. Cube



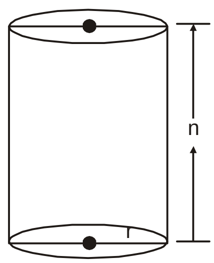
A right prism in which the base is a square and height is equal to the side of the base is alled a cube. If x is the side of the cube, then lateral surface area = $4x \times x = 4x^2$sq unit

Total surface area = $4x^2 + 2(x^2) = 6x^2$ sq unit

Volume = $x^2 \times x = x^3$ cu unit

The longest diagonal of the cube = $\sqrt{3x}$ unit

5. Cylinder



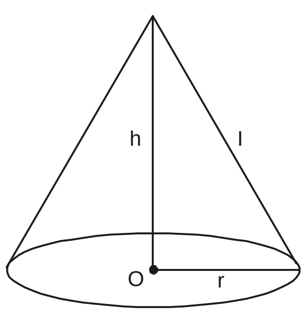
If the base of a right prism is circular, then it becomes a cylinder. The lateral surface area c the cylinder is a single curved surface called the curved surface area. If r is the radius of the has and h is the height of the cylinder, then

Curved surface area = $2\pi rh$ sq unit

Total surface area = $2\pi rh + 2\pi r^2 = 2\pi r\,(r + h)$ sq unit

Volume of cylinder = $\pi r^2 h$ cu unit

6. Cone



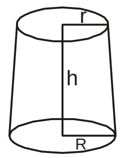
If the base of a right pyramid is circular, then it becomes a cone. The lateral surface area of the cone is a single curved surface. If r is the radius of the base h is the height of the cone and 1 the slant height of the cone, then slant height, $l = \sqrt{h^2 + r^2}$ unit

Curved surface area $= \pi r l$ sq unit

Total surface area $= \pi r l + \pi r^2 = \pi r\,(l + r)$ sq unit

Volume of cylinder $= \dfrac{1}{3}\ \pi r^2 h$ cu unit.

7. Frustrum of a Cone



If a cone is cut parallel to base at a hight h then the remaining part is called the frustrum of the cone.

If radius of base and vertex of the frustrum of a cone be R and r respectively and height and slant height of it be h and l then,

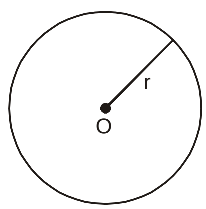
Area of slant part $= \pi(R + r)$ sq.units

where $l^2 = h^2 + (R - r)^2$

Total surface area $= \pi R^2 + \pi r^2 + \pi l(R + r) = \pi\,[R^2 + r^2 + l(R + r)]$ sq.unit

Volume $= \dfrac{1}{3}\pi h\,[R^2 + r^2 + R\mathrm{r}]$ cu unit

8. Sphere

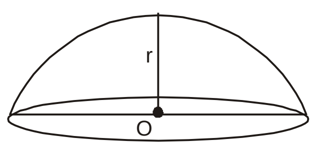


A sphere is a solid in which any point on the surface of sphere is equidistant from the centre of the sphere.

Surface area $= 4\pi r^2$ sq unit

Volume $= \dfrac{4}{3}\pi r^3$ cu unit

9. Hemisphere



When a sphere is cut into two equal halves, each half is an hemisphere in which the base is circular with radius r and height is equal to the radius.

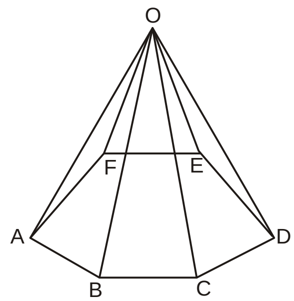
Curved surface area $= 2\pi r^2$ sq. unit

Total surface area $= 2\pi r^2 + \pi r^2$sq. unit $= 3\pi r^2$ sq. unit

Volume $= 2\pi r^3$ cu unit

1 L = 1000cm^3

10. Pyramids



If a point O is joined to the end points of each side of a polygon by straight lines, then it is called a pyramid.

$$\text{Slant surface area} = \frac{1}{2} \times \text{Slant height} \times \text{Base side}$$

$$\text{Total surface area} = \text{Base area} + \text{Slant area}$$

$$\text{Volume} = \frac{1}{3} \times \text{Base area} \times \text{Height}$$

Base area depends on the shape of polygon in the base.

Example 1: Find the volume and total surface area of a cube of side 7 m.

Solution. Here, x =7 m

$$\text{Surface area} = 6 \times 2 = (6 \times 72)\ \text{m}^2 = 294\ \text{m}^2$$
$$\text{Volume} = x^3 = (7^3)\ \text{m}^3 = 343\ \text{m}^3$$

Example 2: Find the volume and total surface area of a cuboid 10 m long, 6 m broad and 4 m high.

Solution. Here, l =10 m, b = 6 m and h = 4 m

$$\text{Volume} = lbh = (10 \times 6 \times 4)\ \text{m}^3 = 240\ \text{m}^3$$
$$\text{Surface area} = 2\,(lb + bh + lh) = 2\,[(10 \times 6) + (6 \times 4) + (4 \times 10)]\ \text{m}^2$$
$$= 2\,[60 + 24 + 40] = 248\ \text{m}^2$$

Example 3: Find the length of the longest pole that can be put in a room 15 m long, 8 m broad and 7 m high.

Solution. Here, l = 15 m, b = 8 m and h = 7 m

$$\text{Required length} = \sqrt{l^2 + b^2 + h^2} = \sqrt{(15)^2 + (8)^2 + (7)^2} = \sqrt{388} = 13\sqrt{2}\ \text{m}$$

Example 4: The length, breadth and height of a room are in the ratio 3 : 2 : 1. The length, breadth and height of the room are increased by 300%, 200% and 100% respectively. Find, how many number of times the volume of the room is increased?

Solution. Let the length, breadth and height of the room are $3x$, $2x$ and x respectively.

$$\text{Volume of the room} = 3x \cdot 2x \cdot x = 6x^2 \text{ After increase}$$
$$\text{length} = 3x + 300\% \text{ of } 3x = 12x$$
$$\text{breadth} = 2x + 200\% \text{ of } 2x = 6x$$
$$\text{height} = x + 100\% \text{ of } x = 2x$$
$$\text{New volume} = 12x + 6x + 2x = 144x^3$$
$$\text{Increase in volume} = 144x^3 - 6x^3 = 138x^3$$

Required number of times $= \dfrac{138x^3}{6x^3}$ = 23 times

Example 5: From a solid sphere of radius 7cm, a right circular cylinderical hole of radius 3 cm and its axis passing through the centre is removed. Find the total surface area of the remaining solid.

Solution. Clearly, height of the cylinder = diameter of the sphere

= 14 cm

Required total surface area $= 4\pi(7)^2 - 2\pi(3)^2 + 2\pi(3) \times (14)$

$= 4\pi \times 49 - 2\pi \times 9 + 2\pi \times 42$

$= 280\pi - 18\pi = 262\pi$

Example 6: The base radius of a cylinder is 14 cm and its height is 30 cm. Find (a) Volume (b) Curved surface area (c) Total surface area

Solution. (a) Volume of cylinder $= \pi r^2 \times h = \dfrac{22}{7} \times 14 \times 30 = 18480\ \text{cm}^3$

(b) Curved surface area $= 2\pi rh = 2 \times \dfrac{22}{7} \times 14 \times 30$

$= 2640\ \text{cm}^3$

(c) Total surface area $= 2\pi r\,(h + r) = 2 \times \dfrac{22}{7} \times 14(30 + 14)$

$= 2 \times \dfrac{22}{7} \times 14 \times 44 = 3872\ \text{cm}^2$

Example 7: How many cubic metres of earth must be dug to make a well 14 m deep and 4 m in diameter?

Solution. Each to be dugout from the well = volume of the cylindrical well

$= \pi r^2 h = \dfrac{22}{7} \times 2 \times 2 \times 14 = 176\ \text{m}^3$

Example 8: The radius of the base of a right cone is 35 cm and its height is 84 cm. Find
(a) Slant Height (b) Curved Surface Area (c) Total Surface Area (d) Voume

Solution. (a) Slant height (l) $= \sqrt{r^2 + h^2}$ ($r \to$ radius of the circular base)

$= \sqrt{35^2 + 84^2}$ ($h \to$ height of the cone)

$= \sqrt{1225 + 7056} = \sqrt{8281} = 91$ cm

(b) Curved surface area $= \pi rl = \dfrac{22}{7} \times 35 \times 91 = 10010\ \text{cm}^2$

(c) Total surface area = lateral surface area + base area

$= \pi rl + \pi r^2 = \pi r(l + r)$

$= \dfrac{22}{7} \times 35(91 + 84)$

$= 110 \times 175 = 19250\ \text{cm}^2$

(d) Volume $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 35 \times 35 \times 84 = 107800 \text{ cm}^3$