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CHAPTER 3

Indices and Surds

In our day-to-day life, we get plenty of data in form of powers of numbers. To have a feeling of what they measure we need to have an exact idea of indices and surds. There may be some kit items in CSAT, specifically designed to test proficiency of candidates in indices and surds.

Indices

In the expression x^n, n is called the exponent or index and x is called the base and x^n is read as 'x to the power of n' or 'x raised to the power n'.

eg,

1. $3^6 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 729$

$4^3 = 4 \times 4 \times 4 = 64$

The expression $(x^m)^n$ is read as 'x raised to the power m whole raised to the power n'.

2. $(2^4)^3 = (16)^3 = 2^{12}$

The expression x^{m^n} is read as 'x raised to the power m power n'.

3. $2^{4^3} = 2^{64}$

Hence, $(x^m)^n \neq x^{m^n}$

Laws of Indices

1. $a^m \times a^n = a^{m+n}$
2. $\dfrac{a^m}{a^n} = a^{m-n}$ $(a \neq 0)$
3. $(a^m)^n = a^{mn}$
4. $(ab)^m = a^m b^m$
5. $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$ (b ¹ 0)
6. $a^0 = 1$ if $(a \neq 0)$
7. $a^{-n} = \dfrac{1}{a^n}$ if $(a \neq 0)$
8. $a^{\frac{1}{n}} = \sqrt[n]{a}$ is called the nth root of a.
9. $a^{\frac{m}{n}} = \sqrt[n]{a^m}$ is called the nth root of a^m.

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10. When the bases of two numbers are equal, then their powers are also equal.
    eg, If $3^n = 3^2$, then, $n = 2$.
11. When the powers of two numbers are equal, then their bases are equal.
    eg, If $a^3 = 43$, then $a = 4$.
    Where a is a positive real number and n is a rational number.

Surds

If n is a positive integer and 'a' is a positive rational number (a > 0), then $\sqrt[n]{a}$ is called a surd of order n or nth order surd if $\sqrt[n]{a}$ is an irrational number.

$$\sqrt[3]{5}, \sqrt[4]{7}, 2+\sqrt{7}, 2-\sqrt{7}, \sqrt{3}+\sqrt{5}, \sqrt{3}-\sqrt{5}$$

1. **Simple Surd:** A surd which consists of a single term is called surd or monomial surd.

   eg, $\sqrt{3}, \sqrt[4]{3}, \sqrt[n]{a}$

2. **Mixed Surd:** If 'a' is a rational number and $\sqrt{b}$ is a surd, then $a+\sqrt{b}, a-\sqrt{b}$ are called mixed surds.

   eg, $2+\sqrt{3}, 2-\sqrt{3}$

3. **Compound Surd:** A surd which is a sum or difference of two or more surds is called a compound surd.

   eg, $2+\sqrt{3}, 2-\sqrt{3}$

4. **Similar Surds:** If two surds are different multiples of the same surd, they are called similar surds otherwise they are dissimilar surds.

   eg, $2\sqrt{3}, 4\sqrt{3}, 5\sqrt{3}$ are similar surds.

   $5\sqrt{2}, 7\sqrt{3}, 3\sqrt{5}$ are dissimilar surds.

Laws of Radicals

(i) $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$

(ii) $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$

(iii) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$

(iv) $\sqrt[n]{a^p} = a^{\frac{p}{n}} = \sqrt[mn]{a^{pm}}$

(v) $\left(\sqrt[n]{a}\right)^n = a$

Rationalisation of a Surd

If the product of two surds is rational, then each of the two surds is called a rationalising factor of the other. In general if the surd is of type $a+\sqrt{b}$, then its rationalising factor is $a-\sqrt{b}$.

The rationalising factor of a surd is not unique. But it is always convenient to use the simplest of all rationalising.

Factors of a given Surd

(a) Rationalisig Factor of $\dfrac{1}{\sqrt{a}} = \sqrt{a}$

(b) Rationalising Factor of $\dfrac{1}{a \pm \sqrt{b}} = a \ m\sqrt{b}$

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(c) Rationalising Factor of $\frac{1}{\sqrt{a} \pm \sqrt{b}} = \sqrt{a}\ \mathrm{m}\sqrt{b}$

'Smart' Facts

1. The symbol $\sqrt[n]{a}$ is called the radical sign.
2. In $\sqrt[n]{a}$, 'n' is called the order of the surd.
3. In $\sqrt[n]{a}$, 'a' is called the radicand.
4. By $\sqrt[n]{a}$ is a surd is understood that 'a' is rational number and $\sqrt[n]{a}$ is an irrational number.

Comparison of Surds

Comparison of surds is possible only when they are of the same order. The radicals can be compared.

eg, $\sqrt[3]{2}$ and $\sqrt[3]{8}$ = $2^{\frac{1}{3}}$ and $8^{\frac{1}{3}}$ and since 8 > 2

$\sqrt[3]{8} > \sqrt[3]{2}$.

If the surds are of different order and different base, we first reduce them to the same order.

eg, $\sqrt[4]{5}$ and $\sqrt[3]{4}$ are $5^{\frac{1}{4}}$ and $4^{\frac{1}{3}}$. The LCM of 4 and 3 is 12.

$\Rightarrow$ $5^{\frac{1}{4}} = 5^{\frac{3}{12}} = (5^3)^{\frac{1}{12}} = \sqrt[12]{125}$

$4^{\frac{1}{3}} = 4^{\frac{4}{12}} = (4^4)^{\frac{1}{12}} = (256)^{\frac{1}{12}} = \sqrt[12]{256}$ and 256 > 125

Hence, $\sqrt[3]{4} > \sqrt[4]{5}$

Example 1: Simplify

(a) $(128)^{\frac{3}{7}}$ (b) $2^{25} \times 2^{35} - 2^{20} \times 2^{40}$

Solution.

(a) $(128)^{\frac{3}{7}} = (2^7)^{\frac{3}{7}} = 2^3 = 8$ (b) $2^{25} \times 2^{35} - 2^{20} \times 2^{40} = 2^{60} - 2^{60} = 0$

Example 2. If $a = 3$ and $b = 2$, then find the values of

(a) $a^a + b^b$ (b) $\left(\frac{1}{a} + \frac{1}{b}\right)^a$

Solution. (a) $a^a + b^b = 3^3 + 2^2 = 27 + 4 = 31$ (b) $\left(\frac{1}{a} + \frac{1}{b}\right)^a = \left(\frac{1}{3} + \frac{1}{2}\right)^3 = \left(\frac{5}{6}\right)^3 = \frac{125}{216}$

Example 3. Evaluate

(a) $(64)^{-\frac{1}{3}}$ (b) $(125)^{-\frac{2}{3}}$ (c) $\sqrt[5]{32^{-3}}$

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Solution.

(a) $(64)^{-\frac{1}{3}} = \frac{1}{(64)^{\frac{1}{3}}} = \frac{1}{(4^3)^{\frac{1}{3}}} = \frac{1}{4}$

(b) $(125)^{-\frac{2}{3}} = \frac{1}{(125)^{\frac{2}{3}}} = \frac{1}{(5^3)^{\frac{2}{3}}} = \frac{1}{5^2} = \frac{1}{25}$

(c) $\sqrt[5]{32^{-3}} = (2^5)^{\frac{3}{5}} = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

Example 4: If $2^{x-7} . 5^{x-4} = 1250$. Find x.

Solution. We have $2^{x-7} . 5^{x-4} = 1250$

$\Rightarrow$ $2^{x-7} . 5^{x-4} = 2.625 = 2^1 \times 5^4 \Rightarrow x - 7 = 1$ and $x - 4 = 4$

$\Rightarrow$ $x = 8$ and $x = 8$.

Thus, $x = 8$

Example 5: Simplify $\frac{1}{1+x^{b-a}} + \frac{1}{1+x^{a-b}}$.

Solution. Given expression $= \frac{1}{1+x^{b-a}} + \frac{1}{1+x^{a-b}} = \frac{x^{-b}}{x^{-b}(1+x^{b-a})} + \frac{x^{-a}}{x^{-a}(1+x^{a-b})}$

$= \frac{x^{-b}}{x^{-b}+x^{-a})} + \frac{x^{-a}}{x^{-a}+x^{-b})} = \frac{x^{-1}+x^{-a}}{x^{-a}+x^{-b}} = 1$

Example 6: Simplify the following expressions.

(a) $(3+\sqrt{3})(2-\sqrt{2})$

(b) $(3+\sqrt{5})(2+\sqrt{2})$

Solution. Given (a) $(3+\sqrt{3})(2-\sqrt{2}) = 3\times 2 - 3\times\sqrt{2} + \sqrt{3}\times 2 - \sqrt{3}\times\sqrt{2} = 6 - 3\sqrt{2} + 2\sqrt{3} - \sqrt{6}$

(b) $(3+\sqrt{5})(2+\sqrt{2}) = 6 + 3\sqrt{2} + 2\sqrt{5} + \sqrt{10}$

Example 7: Rationalise the denominator of

(a) $\frac{1}{3+4\sqrt{2}}$

(b) $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

Solution.

(a) $\frac{1}{6+4\sqrt{2}} = \frac{1}{6+4\sqrt{2}} \times \frac{6-4\sqrt{2}}{6-4\sqrt{2}} = \frac{6-4\sqrt{2}}{(6)^2-(4\sqrt{2})^2}$

(b) $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{(\sqrt{5}+\sqrt{3})^2}{(\sqrt{5})^2-(\sqrt{3})^2} = \frac{5+3+2\sqrt{15}}{5-3} = \frac{8+2\sqrt{15}}{2} = 4+\sqrt{15}$

Example 8: Arrange the following in ascending order of magnitude $\sqrt[4]{10}, \sqrt[3]{6}, \sqrt{3}$.

Solution. Given, $\sqrt[4]{10}, \sqrt[3]{6}, \sqrt{3}$

$\sqrt[4]{10}$ is a surd of order 4

$4\sqrt[3]{6}$ is a surd of order 3

$\sqrt{3}$ is a surd of order 2 $\therefore$ LCM of 4, 3, 2 is □ 12

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$$\sqrt[4]{10} = 10^{\frac{1}{4}} = (10)^{\frac{3}{12}} = \sqrt[12]{10^3} = \sqrt[12]{1000}$$

$$\sqrt[3]{6} = 6^{\frac{1}{3}} = (6)^{\frac{4}{12}} = \sqrt[12]{6^4} = \sqrt[12]{1296}$$

and $$\sqrt{3} = 3^{\frac{1}{2}} = (3)^{\frac{6}{12}} = \sqrt[12]{3^6} = \sqrt[12]{729}$$

$\therefore$ The ascending order is $\sqrt[12]{729}, \sqrt[12]{1000}, \sqrt[12]{1296}$ ie, $\sqrt{3}, \sqrt[4]{10}, \sqrt[3]{6}$

Example 9: Find the square root of $13+2\sqrt{40}$.

Solution. $\sqrt{13+2\sqrt{40}} = \sqrt{a}+\sqrt{b} \Rightarrow 13+2\sqrt{40} = a + b + 2\sqrt{ab}$

$\Rightarrow$ $a + b = 13$ and $ab = 40$

$\Rightarrow$ $(a - b)^2 = (a + b)^2 - 4ab = (13)^2 - 4 \times 40 = 169 - 160 = 9$

$\Rightarrow$ $a - b = 3$ and $a + b = 13$

Hence, $a = 8$ and $b = 5$ $\therefore$ $\sqrt{13+2\sqrt{40}} = \sqrt{8}+\sqrt{5}$

Example 10: Simplify $\sqrt[4]{81} - 8\sqrt[3]{216} + 15\sqrt[5]{32} + \sqrt{225}$

Solution. $\sqrt[4]{(3)^4} - 8\sqrt[3]{6^3} + 15\sqrt[5]{2^5} + \sqrt{15\times15} = (3^{\frac{4}{4}}) - 8(6^{\frac{3}{3}}) + 15(2^{\frac{5}{5}}) + 15 = 3 - 8 \times 6 + 15 \times 2 + 15$

$= 3 - 48 + 30 + 15 = 48 - 48 = 0$

EXERCISE

1. The expression $\sqrt{\dfrac{.85 \times (.105 + .024 - .008)}{.022 \times .25 \times 1.7}}$ simplifies to:

(a) $\sqrt{11}$ (b) $\sqrt{1.1}$

(c) $\sqrt{0.11}$ (d) $\sqrt{.011}$

2. The value of $\sqrt{\dfrac{16}{36}+\dfrac{1}{4}}$ is:

(a) 4/5 (b) 1/3

(c) 5/6 (d) 8/15

3. A decimal number has 16 decimal places. The number of decimal places in the square root of this number will be:

(a) 7 (b) 4 (c) 8 (d) 16

4. Consider the following values of three given numers:

$\sqrt{103}, \sqrt{99.35}, \sqrt{102.20}$

(a) 10.1489 (Approx.) (b) 10.109 (Approx.)

(c) 9.967 (Approx.)

The correct sequence of these values matching with the above number is:

(a) 3, 2, 1 (b) 1, 3, 2

(c) 2, 3, 1 (d) 3, 1, 2

5. If a four-digited perfect square number is such that the number formed by the first two digits and the number formed by the last two digits are also perfect squares, then the four digited number is:

(a) 5625 (b) 3616

(c) 1681 (d) 1024

6. $\sqrt{6+\sqrt{6+\sqrt{6+....}}}$ is equal to:

(a) $6\frac{3}{2}$ (b) $3\frac{1}{2}$ (c) 6 (d) 3

7. The value of the expression,

$\sqrt{4+\sqrt{15}} + \sqrt{4-\sqrt{15}} - \sqrt{12-\sqrt{4\sqrt{5}}}$

(a) infinite number

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(b) natural number
(c) a negative integer (d) an irrational number

8. Evaluate $\sqrt{41-\sqrt{21+\sqrt{19-\sqrt{9}}}}$:

(a) 4 (b) 8
(c) 6 (d) 6.4

9. $\sqrt{1+\sqrt{1+\sqrt{1+\ldots}}}$:

(a) equals 1
(b) lies between 0 and 1
(c) lies between 1 and 2
(d) is greater than 2

10. If $\frac{52}{x} = \sqrt{\frac{169}{289}}$, the value of x:

(a) 68 (b) 64
(c) 78 (d) 52

11. The simplified value of

$$\frac{\sqrt{8+\sqrt{28}}-\sqrt{8-\sqrt{28}}}{\sqrt{8+\sqrt{28}}+\sqrt{8-\sqrt{28}}} \text{ is:}$$

(a) $\frac{1}{\sqrt{7}}$ (b) $\frac{2}{\sqrt{7}}$ (c) $3\sqrt{7}$ (d) $\frac{1}{2\sqrt{7}}$

12. $\frac{(225)^{0.2} \times (225)^{0.3}}{(225)^{0.8} \times (225)^{0.2}}$ is equal to:

1. 2/15 2. 1/15 3. 1/25
4. 3.5 5. 15^2

13. $\left(\frac{1}{64}\right)^{0} + (64)^{-\frac{1}{2}} + (32)^{\frac{4}{5}} - (32)^{-\frac{4}{5}}$ is equal to:

(a) $17\frac{3}{16}$ (b) $17\frac{3}{10}$

(c) $17\frac{1}{16}$ (d) $-17\frac{1}{16}$

14. The value of

$$\sqrt{10+\sqrt{25+\sqrt{108+\sqrt{154+\sqrt{225}}}}} \text{ is:}$$

(a) 3 (b) 9
(c) 6 (d) 4

15. The simplified value of

$$\frac{(0.55)^2+(0.07)^2+(0.027)^2}{(0.055)^2+(0.007)^2+(0.0027)^2} \text{ is:}$$

(a) 1 (b) 100 (c) 10000 (d) 0.1

16. On simplifications of

$$\frac{0.01 \times 0.01 \times 0.01+0.09 \times 0.09 \times 0.09}{0.01 \times 0.01-0.0009+0.09 \times 0.09},$$

we get:

(a) 1.0 (b) 0.2
(c) 4 (d) 0.12

17. What is the sum of the squares of the following numbers?

$$\frac{\sqrt{3}}{\sqrt{2+1}}, \frac{\sqrt{3}}{\sqrt{2-1}}, \frac{\sqrt{2}}{\sqrt{3}}$$

(a) $18\frac{1}{3}$ (b) $16\frac{2}{3}$ (c) 20 (d) $18\frac{2}{3}$

18. Two numbers are in the ratio of 9 : 7. If the difference of their squares is 288, then the smaller of the number is:

(a) 21 (b) 23
(c) 27 (d) 28

19. On simplification of $\frac{(4.7)^3+0.027}{(4.7)^2-1.41+0.09}$ we get:

(a) 7 (b) 1.41
(c) 5 (d) 6

20. What value should come in place of the question mark (?) in the following equation?

$48\sqrt{?}+32\sqrt{?}=320$

(a) 25 (b) 16
(c) 2 (d) 4

21. $\left(\frac{64}{25}\right)^{-\frac{3}{2}} = ?$

(a) $\frac{125}{215}$ (b) $\frac{125}{512}$

(c) $\frac{125}{72}$ (d) None of these

22. $(0.001)^{\frac{1}{3}} = ?$

(a) 0.1 (b) 0.01
(c) 1.1 (d) None of these

23. If $27x = \frac{9}{3^x}$, then the value of x is

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(a) $\frac{1}{2}$ (b) 1
(c) $\frac{3}{2}$ (d) None of these

24. The value of $\frac{5^n + 5^{n-1}}{5^{n+1} - 5^n} = ?$
(a) 0.1 (b) 0.2
(c) 0.3 (d) None of these

25. If $-2^{x-7} \times 5^{x-4} = 1250$, then $x = ?$
(a) 2 (b) 4
(c) 6 (d) 8

26. $\frac{x^{a(b-c)}}{x^{b(a-c)}} \div \left(\frac{x^b}{x^a}\right)^c = ?$
(a) 1 (b) 2
(c) 3 (d) None of these

27. $\frac{(x^{a+b})^2 (x^{b+c})^2 (x^{c+a})^2}{(x^a x^b x^c)^4} = 1$
(a) 1 (b) 2
(c) 3 (d) None of these

28. Find the value of x, if $5^{x-3} \cdot 3^{2x-8} = 225$.
(a) 3 (b) 4
(c) 5 (d) None of these

29. If $K = \sqrt[3]{54} + \sqrt[3]{128} + \sqrt[3]{250} - \sqrt[3]{16}$, then the value of K = ?
(a) 10 (b) $10(3)^{\frac{1}{3}}$
(c) $10(2)^{\frac{1}{3}}$ (d) None of these

30. Square root of $15 + 4\sqrt{14} = ?$
(a) $\sqrt{6} + \sqrt{9}$ (b) $\sqrt{7} + \sqrt{8}$
(c) $\sqrt{10} + \sqrt{5}$ (d) None of these

31. $\frac{5\sqrt{3} + 2\sqrt{2}}{2\sqrt{3} - 3\sqrt{2}} = ?$
(a) $42 + 19\sqrt{6}$ (b) $\frac{42 + 19\sqrt{6}}{-6}$
(c) $42 - 19\sqrt{6}$ (d) None of these

32. If $x = 7 - 4\sqrt{3}$, then the value of $\sqrt{x} + \frac{1}{\sqrt{x}} = ?$
(a) 7 (b) −7 (c) 4 (d) −4

33. If $\sqrt{a + \sqrt{b}} = 2 + \sqrt{3}$, then find $\sqrt{a^2 - b}$.
(a) 3 (b) 2
(c) 1 (d) None of these

34. Which of the following is greatest $\sqrt[3]{2}, 2\sqrt[3]{4}, 2\sqrt{3}$?
(a) $\sqrt[3]{2}$ (b) $2\sqrt[3]{4}$
(c) $2\sqrt{3}$ (d) None of these

35. If $x = 2 + 2^{\frac{1}{3}} + 2^{\frac{2}{3}}$, then $x^3 - 6x^2 + 2 = ?$.
(a) 0 (b) 2 (c) 4 (d) 8

36. If $\frac{4 + 3\sqrt{3}}{\sqrt{7 + 4\sqrt{3}}} = a + \sqrt{b}$, then $(a, b) = ?$
(a) (12, 1) (b) (1, 12)
(c) (−1, 12) (d) (−12, 1)

37. If $\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c} = 0$, then $(a + b + c)^3 = ?$
(a) 3 abc (b) 9 abc
(c) 27 abc (d) None of these

38. The value of $\frac{\sqrt{7}}{\sqrt{16 + 6\sqrt{7} - \sqrt{16 - 6\sqrt{7}}}}$ is
(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2

39. If $2^x = 3^y = 12^z$, then $\frac{1}{z} - \frac{1}{y} = ?$
(a) $\frac{2}{x}$ (b) $\frac{1}{x}$ (c) $-\frac{1}{x}$ (d) $-\frac{2}{x}$

40. If $\frac{x}{b^2 - c^2} = \frac{y}{c^2 - a^2} = \frac{z}{a^2 - b^2}$, then
$\frac{x}{b + c} + \frac{y}{c + a} + \frac{z}{a + b} = ?$
(a) 1 (b) 0
(c) 2 (d) None of these

41. If $2^a = 4^b = 8^c$ and $abc = 288$, then $\frac{1}{2a} + \frac{1}{4b} + \frac{1}{6c} = ?$
(a) $\frac{1}{12}$ (b) $\frac{1}{2}$ (c) $\frac{1}{8}$ (d) $\frac{1}{24}$

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42. $\dfrac{\sqrt{\frac{5}{2}}+\sqrt{7-3\sqrt{5}}}{\sqrt{\frac{7}{2}}+\sqrt{16-5\sqrt{7}}}=?$

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$
(c) $\frac{3}{5}$ (d) None of these

43. If $x+\frac{1}{x}=\sqrt{3}$, then $x^2+\frac{1}{x^2}=?$

(a) 14 (b) 0
(c) 1 (d) None of these

44. If $x+\frac{1}{x}=\sqrt{3}$, then $x^3+\frac{1}{x^3}=?$

(a) 9 (b) 0
(c) 1 (d) None of these

45. If $x=\frac{\sqrt{2}+1}{\sqrt{2}-1}$ and $y=\frac{\sqrt{2}-1}{\sqrt{2}+1}$, then $3(xy+1)=?$

(a) $x+y$ (b) $x-y$
(c) $2(x+y)$ (d) None of these

46. If $y=\frac{\sqrt{x+a}-\sqrt{x-a}}{\sqrt{x+a}+\sqrt{x-a}}$, then $y+\frac{1}{y}=?$

(a) $\frac{x}{a}$ (b) $\frac{2x}{a}$
(c) $\frac{2a}{x}$ (d) None of these

47. $x=\sqrt{3}+\sqrt{2}$, then $\frac{x+1}{x-1}-\frac{x-1}{x+1}$ is

(a) $\sqrt{3}$ (b) $\sqrt{2}$
(c) $\sqrt{6}$ (d) None of these

48. If $\frac{\left(\sqrt{2+1}\right)^2}{3-\sqrt{2}}=a+b\sqrt{2}$, then $a+b=?$

(a) $\frac{21}{7}$ (b) $\frac{22}{7}$
(c) $\frac{23}{7}$ (d) None of these

49. If $\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}+\frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}=a\sqrt{5}+b\sqrt{6}$, then $a+b=?$

(a) 0 (b) 1
(c) 2 (d) None of these

50. $a=\frac{\sqrt{3}}{2}$, then $\sqrt{1+a}+\sqrt{1-a}=?$

(a) a (b) $2a$
(c) $3a$ (d) None of these

51. If $x=a^p$, $y=a^q$ and $x^q y^p=a^{\frac{2}{r}}$, then $pqr=?$

(a) 1 (b) 2
(c) 3 (d) 4

52. $\frac{1}{1+x^{a-b}}+\frac{1}{1+x^{b-a}}=?$

(a) 0 (b) 1
(c) 2 (d) None of these

53. If $x=\frac{2}{3+\sqrt{7}}$, then $(x-3)^2=?$

(a) 1 (b) 3
(c) 7 (d) 6

54. The square root of $8+2\sqrt{15}$, is

(a) $\sqrt{5}+\sqrt{3}$ (b) $\sqrt{5}-\sqrt{3}$
(c) $\sqrt{5}+3$ (d) $5+\sqrt{3}$

55. If $x=2\sqrt{2}+\sqrt{7}$, then $x+\frac{1}{x}=?$

(a) $2\sqrt{2}$ (b) $4\sqrt{2}$
(c) 8 (d) 7

56. $\sqrt{2+\sqrt{2+\sqrt{2+\ldots\infty}}}=?$

(a) 1 (b) 2
(c) ± 2 (d) None of these

57. If $a=\sqrt{7}-\sqrt{5}$ and $b=\sqrt{13}-\sqrt{11}$, then

(a) $a>b$ (b) $a<b$
(c) $a=b$ (d) None of these

58. $8^{\frac{2}{3}}-16^{\frac{1}{4}}+9^{\frac{1}{2}}$ is equl to

(a) 3 (b) 5
(c) $\sqrt{2}$ (d) $\sqrt{2}+\sqrt{3}$

59. If $9^{2x-1}=27^{x+2}$, then x is equal to

(a) 2 (b) 4
(c) 8 (d) 16

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60. If $a + b + c = 0$, then the value of

$$x^{\left(\frac{a^5}{3a^3bc}\right)} x^{\left(\frac{b^5}{3ab^3c}\right)} x^{\left(\frac{c^5}{3abc^3}\right)} \text{ is}$$

(a) x^3 (b) x^2 (c) x (d) 1

61. Which of the following is the greatest?
(a) 3333^{33} (b) 333^{333}
(c) 33^{3333} (d) 3^{33333}

62. If $\left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}} = 2^x$, then x is equal to
(a) 4 (b) 2
(c) 1 (d) $\sqrt{2}$

63. $\dfrac{5^{23}\times\sqrt[3]{5^8}}{\sqrt[3]{5^7}} = ?$

(a) 5 (b) 25 (c) $\sqrt[3]{5}$ (d) 1

64. $\dfrac{5\times(2^{k-2})+10\times(2^{k-1})}{10^{k+2}} = ?$

(a) $\dfrac{1}{8\times(5^k)}$ (b) $\dfrac{1}{16\times(5^k)}$

(c) $\dfrac{1}{32\times(5^k)}$ (d) $\dfrac{1}{4\times(5^k)}$

65. $\left(\dfrac{\sqrt[4]{ab}-\sqrt{b}}{\sqrt{a}-\sqrt[4]{ab}}\right)^{-4} = ?$

(a) $\dfrac{a^2}{b^2}$ (b) $-\dfrac{a}{b}$ (c) $\sqrt{\dfrac{a}{b}}$ (d) $\dfrac{a}{b}$

66. If $3^{x+1} + 3^{2x+1} = 270$, then x is equal to
(a) 1 (b) 2
(c) 0 (d) 4

67. Determine a possible value of x from the equation $5^{x-2} \times 2^{3x/x+1} = 4$.
(a) 0 (b) 4
(c) 2 (d) 3

68. The least value of $3^x + 3^{-x}$ is
(a) 2 (b) 1
(c) 0 (d) 2/3

69. Given $t = 2+\sqrt[3]{4}+\sqrt[3]{2}$, determine the value of $t^3 - 6t^2 + 6t - 2$.
(a) 0 (b) 1
(c) $\sqrt{2}$ (d) 2

70. If $a = -5$ and $b = -6$, find the value of $(a - b)^{b-a} + (b - a)^{a-b}$.
(a) 1 (b) –1
(c) 0 (d) –2

71. In Republic Day Parade, 6440 NCC cadets were asked to stand in rows to form a perfect square. It was fond that 40 NCC cadets were left out. What was the number of NCC cadets in each row?
(a) 40 (b) 80
(c) 64 (d) 60

ANSWERS

1. (a)	**2.** (c)	**3.** (c)	**4.** (b)	**5.** (c)	**6.** (d)	**7.** (d)	**8.** (c)	**9.** (c)	**10.** (a)
11. (a)	**12.** (b)	**13.** (c)	**14.** (d)	**15.** (b)	**16.** (d)	**17.** (d)	**18.** (a)	**19.** (c)	**20.** (b)
21. (b)	**22.** (a)	**23.** (a)	**24.** (c)	**25.** (d)	**26.** (a)	**27.** (a)	**28.** (c)	**29.** (c)	**30.** (b)
31. (b)	**32.** (c)	**33.** (c)	**34.** (c)	**35.** (c)					
36. (*)	**37.** (c)	**38.** (b)	**39.** (a)	**40.** (b)					
41. (c)	**42.** (c)	**43.** (c)	**44.** (b)	**45.** (a)					
46. (b)	**47.** (b)	**48.** (b)	**49.** (a)	**50.** (b)					
51. (a)	**52.** (b)	**53.** (c)	**54.** (a)	**55.** (b)					
56. (b)	**57.** (a)	**58.** (b)	**59.** (c)	**60.** (c)					
61. (d)	**62.** (c)	**63.** (a)	**64.** (b)	**65.** (d)					
66. (b)	**57.** (c)	**68.** (a)	**69.** (a)	**70.** (c)					
71. (b)									

EXPLANATIONS

1. $\sqrt{\dfrac{0.85\times(0.105+0.024-0.008)}{0.022\times2.25\times1.7}}$

$= \sqrt{\dfrac{0.85\times0.121}{0.022\times2.25\times1.7}} = \sqrt{\dfrac{85\times121\times10}{22\times25\times17}}$

$= \sqrt{11}$

2. $\sqrt{\dfrac{16}{36}+\dfrac{1}{4}} = \sqrt{\dfrac{4}{9}+\dfrac{1}{4}} = \sqrt{\dfrac{16+9}{36}}$

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$$= \sqrt{\frac{25}{36}} = \frac{5}{6}$$

3. The number of decimal places in the square root of a number is half of the number of decimal places in that number.

4. $\sqrt{103} = 10.1489$, $\sqrt{99.35} = 9.967$

$= \sqrt{102.20} = 10.109$

5. $\sqrt{1681} = 41$

6. $\sqrt{6+\sqrt{6+\sqrt{6+\ldots}}} = 3$

Let $y = \sqrt{6+\sqrt{6+\ldots\infty}}$

or $y = \sqrt{6+y}$

Squaring both the sides, we get

$y^2 = 6 + y$

or $y^2 - y - 6 = 0$

or $(y-3)(y+2) = 0$

or $y = 3, -2$

$\Rightarrow$ $y = 3$ (leaving –ive sign)

8. $\sqrt{41-\sqrt{21+\sqrt{19-9}}}$

$= \sqrt{41-\sqrt{21+\sqrt{(19-3)}}} = \sqrt{41-\sqrt{21+4}}$

$= \sqrt{41-5} = \sqrt{36} = 6$

9. $y = \sqrt{1+\sqrt{1+\sqrt{1+\ldots\infty}}}$

$\Rightarrow$ $y^2 = 1+\sqrt{1+\sqrt{1+\ldots\infty}}$

$\Rightarrow$ $y^2 - y - 1 = 0$

$\Rightarrow$ $y = \dfrac{1\pm\sqrt{1-4(-1)}}{2}$

$\Rightarrow$ $y = \dfrac{1\pm\sqrt{5}}{2} = \dfrac{1\pm 2.236}{2}$

$[\because \sqrt{5} = 2.236]$

$\Rightarrow y = 1.618$

$\therefore$ y lies between 1 and 2.

10. $\dfrac{52}{x} = \sqrt{\dfrac{169}{289}} = \dfrac{13}{17} \Rightarrow x = \dfrac{52\times 17}{13} = 68$

11. $\dfrac{\sqrt{8+\sqrt{28}}-\sqrt{8-\sqrt{28}}}{\sqrt{8+\sqrt{28}}+\sqrt{8-\sqrt{28}}} \times \dfrac{\sqrt{8+\sqrt{28}}-\sqrt{8-\sqrt{28}}}{\sqrt{8+\sqrt{28}}-\sqrt{8-\sqrt{28}}}$

$$= \frac{\left[\sqrt{8+\sqrt{28-\sqrt{8-28}}}\right]^2}{\left(\sqrt{8-28}\right)^2+\left(\sqrt{8-\sqrt{28}}\right)^2}$$

$$= \frac{\left(8+\sqrt{28}\right)+\left(8-\sqrt{28}\right)-2\sqrt{8+\sqrt{28}}\times\sqrt{8-\sqrt{28}}}{\left(8+\sqrt{28}\right)-\left(8-\sqrt{28}\right)}$$

$$= \frac{16-2\sqrt{64-28}}{2\sqrt{28}} = \frac{16-2\times 6}{2\sqrt{28}} = \frac{4}{2\sqrt{28}}$$

$$= \frac{1}{\sqrt{7}}$$

12. $\dfrac{(225)^{0.2}\times(225)^{0.3}}{(225)^{0.8}\times(225)^{0.2}} = (225)^{0.2+03-0.8+0.2}$

$= (225)^{0.5-1.0} = (225)^{-0.5}$

$= 15^{2\times\left(-\frac{1}{2}\right)} = \dfrac{1}{15}$

13. $\left(\dfrac{1}{64}\right)^0 + (64)^{-1/2} + (32)^{-4/5}$

$= 1 + 8^{2\times-\frac{1}{2}} + 2^{5\times\frac{4}{5}} - 2^{5\times\frac{-4}{5}}$

$= 1 + 8^{-1} + 2^4 - 2^{-4}$

$= 1+\dfrac{1}{8}+16-\dfrac{1}{16} = 17 = 17+\dfrac{1}{16} = 17\dfrac{1}{16}$

14. $\sqrt{10+\sqrt{25+\sqrt{108+\sqrt{154+\sqrt{225}}}}}$

$= \sqrt{10+\sqrt{25+\sqrt{108+\sqrt{(154+15)}}}}$

$= \sqrt{10+\sqrt{25+\sqrt{108+13}}}$

$= \sqrt{10+\sqrt{25+11}} = \sqrt{10+6} = 4$

15. $\dfrac{(0.55)^2+(0.07)^2+(0.027)^2}{(0.055)^2+(0.007)^2+(0.0027)^2}$

$$= \frac{(0.55)^2+(0.07)^2+(0.027)^2}{\frac{1}{10^2}\left[(0.55)^2+(0.07)^2+(0.027)^2\right]} = 100$$

16. $\dfrac{0.01\times 0.01\times 0.01+0.09\times 0.09\times 0.09}{0.01\times 0.01-0.0009+0.09\times 0.09}$

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$$= \frac{(0.01)^3 + (0.09)^3}{(0.01)^2 - 0.01 + 0.09 + (0.09)^2}$$

$$= \frac{(0.01+0.09)(0.01)^2 - 0.01\times 0.09 + (0.09)^2}{(0.01)^2 - 0.01\times 0.09 + (0.09)^2} = 0.1$$

[Q $(a^3 + a^3 = (a + b)(a^2 - ab + b^2)$]

17. $\left(\frac{\sqrt{3}}{\sqrt{2}+1}\right)^2 + \left(\frac{\sqrt{3}}{\sqrt{2}-1}\right)^2 + \left(\sqrt{\frac{2}{3}}\right)^2$

$$= \frac{3}{3+2\sqrt{2}} + \frac{3}{3-2\sqrt{2}} + \frac{2}{3}$$

$$= 3\left[\frac{3-2\sqrt{2}+3+2\sqrt{2}}{9-8}\right] + \frac{2}{3} = 3\times 6 + \frac{2}{3}$$

$$= 18\frac{2}{3}$$

18. Let the numbers be $9x$ and $7x$.

$\therefore \quad 81x^2 - 49x^2 = 288$

or, $\quad 32x^2 = 288$

or, $\quad x^2 = 9$

or, $\quad x = 3$

The smaller number is 21.

19. $\frac{(4.7)^3 + 0.027}{(4.7)^2 - 141 + 0.09}$

$$= \frac{(4.7)^3 + (0.3)^3}{(4.7)^2 - 4.7 + 0.3 + (0.3)^2} = (4.7 + 0.3)$$

$= 5$

20. $48\sqrt{x} + 32\sqrt{x} = 320$

$\Rightarrow \quad (48+32)\sqrt{x} = 320$

$\Rightarrow \quad \sqrt{x} = 4$

$\Rightarrow \quad x = 16$

21. $\left(\frac{64}{25}\right)^{-3/2} = \frac{1}{\left(\frac{64}{25}\right)^{3/2}} = \left(\frac{25}{64}\right)^{3/2}$

$$= \left[\left(\frac{5}{2}\right)^2\right]^{3/2} = \left(\frac{5}{8}\right)^3 = \frac{125}{512}$$

22. $(0.001)^{\frac{1}{3}} = \left(\frac{1}{1000}\right)^{\frac{1}{3}} = \left(\left(\frac{1}{10}\right)^3\right)^{\frac{1}{3}} = \frac{1}{10} = 0.1$

23. Give expression $27^x = \frac{9}{3^x}$

$\Rightarrow \quad (3^3)^x \times 3^x = 9$

$\Rightarrow 3^{4x} = 3^2 \Rightarrow 4^x = 2 \Rightarrow x = \frac{1}{2}$

24. $\frac{5^n + 5^{n-1}}{5^{n+1} - 5^n} = \frac{5^{n-1}(5+1)}{5^n(5-1)} = \frac{5^{n-1}\times 6}{5^n \times 4} = \frac{5^n \times 5^{-1} \times 6}{5^n \times 4}$

$$= \frac{6}{5\times 4} = \frac{3}{10} = 0.3$$

25. The given expression $2^{x-7} \times 5^{x-4} = \frac{2^x}{2^7} \times \frac{5^x}{5^4} = 1250$

$\Rightarrow \quad \frac{(10)^x}{(10)^4 \times 2^3} = 1250$

$\Rightarrow \quad \frac{(10)^x}{(10)^4} = 10000$

$\Rightarrow \quad (10)^{x-4} = (10)^4$

$\Rightarrow \quad x - 4 = 4$

$\Rightarrow \quad x = 8$

26. $\frac{x^{ab-ac}}{x^{ab-bc}} \div \left(\frac{x^{bc}}{x^{ac}}\right) = x^{ab-bc-ab+bc} \times \frac{x^{ac}}{x^{bc}}$

$$= \frac{x^{bc}}{x^{ac}} \times \frac{x^{ac}}{x^{bc}} = 1$$

27. Given expression

$$\frac{(x^{2a+2b})(x^{2b+2c})(x^{2c+2a})}{x^{4a}\times x^{4b}\times x^{4c}} = \frac{x^{4a}x^{4b}x^{4c}}{x^{4a}x^{4b}x^{4c}} = 1$$

28. $5^{x-3} \times 3^{2x-8} = 225$

$\Rightarrow \quad 5^{x-3} \times 3^{2x-8} = 5^2 \times 3^2$

$\Rightarrow \quad x - 3 = 2$ and $2x - 8 = 2$

In any case $x = 5$

29. Give expression $\sqrt[3]{2}[\sqrt[3]{27} + \sqrt[3]{64} + \sqrt[3]{125} - \sqrt[3]{8}]$

$$= \sqrt[3]{2}[(27)^{\frac{1}{3}} + (64)^{\frac{1}{3}} + (125)^{\frac{1}{3}} - (8)^{\frac{1}{3}}]$$

$$= \sqrt[3]{2}[3+4+5-2] = \sqrt[3]{2}[10] = (10)(2)^{\frac{1}{3}}$$

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30. $\sqrt{15+4\sqrt{14}} = \sqrt{15+2\sqrt{56}} = \sqrt{15+2\sqrt{8\times 7}}$

$= \sqrt{(\sqrt{7}+\sqrt{8})^2} = \sqrt{7}+\sqrt{8}$

31. Given expression

$$\frac{5\sqrt{3}+2\sqrt{2}}{2\sqrt{3}-3\sqrt{2}} \times \frac{2\sqrt{3}+3\sqrt{2}}{2\sqrt{3}+3\sqrt{2}} = \frac{30+15\sqrt{6}+4\sqrt{6}+12}{(2\sqrt{3})^2-(3\sqrt{2})^2}$$

$$\Rightarrow \frac{42+19\sqrt{6}}{12-18} = \frac{42+19\sqrt{6}}{-6}$$

32. Given $x = 7-4\sqrt{3}$

$$\Rightarrow x+\frac{1}{x} = (7-4\sqrt{3})+\frac{1}{7-4\sqrt{3}}$$

$= 7-4\sqrt{3}+7+4\sqrt{3} = 14$

Adding 2 on both sides

$$x+\frac{1}{x}+2 = 16 \Rightarrow \left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 = 16$$

$$\Rightarrow \sqrt{x}+\frac{1}{\sqrt{x}} = 4$$

33. $\sqrt{a+\sqrt{b}} = 2+\sqrt{3}$

$\Rightarrow a+\sqrt{b} = ((2+\sqrt{3})^2$

$= 4+3+4\sqrt{3}$

$= 7+4\sqrt{3} = 7+\sqrt{48}$

$\Rightarrow a = 7,\ b = 48$

$\therefore \sqrt{a^2-b} = \sqrt{49-48} = \sqrt{1} = 1$

34. $\sqrt[3]{2}, 2\sqrt[3]{4}, 2\sqrt{3}$

$\sqrt[3]{2}, \sqrt[3]{32}, \sqrt{12}$

LCM of 3 and 2 is 6.

$\sqrt[3]{2} = 2^{\frac{1}{3}} = 2^{\frac{5}{6}} = \sqrt[6]{2^2} = \sqrt[6]{4}$

$\sqrt[3]{32} = (32)^{\frac{1}{3}} = (32)^{\frac{2}{6}} = \sqrt[6]{(32)^2} = \sqrt[6]{1024}$

$\sqrt{12} = (12)^{\frac{1}{2}} = (12)^{\frac{3}{6}} = \sqrt[6]{(12)^3} = \sqrt[6]{1728}$

$\therefore$ Greatest among all is $\sqrt[6]{1728} = \sqrt{12} = 2\sqrt{3}$

35. Given $x = 2+2^{\frac{1}{3}}+2^{\frac{2}{3}}$

$\Rightarrow x-2 = 2^{\frac{1}{3}}(1+2^{\frac{1}{3}})$

$\Rightarrow (x-2)^3 = 2(1+2^{\frac{1}{3}})^3$

$\Rightarrow x^3-6x^2+12x-8 = 2[1+2+3\times 2^{\frac{1}{3}}(1+2^{\frac{1}{3}})]$

$\Rightarrow 2[3+3(x-2)] = 6x-6$

$\Rightarrow x^3-6x^2+6^x-2 = 0$

$\Rightarrow x^3-6x^2+6^x+2 = 4$

36.

$$\frac{4+3\sqrt{3}}{\sqrt{7+4\sqrt{3}}} = \frac{4+3\sqrt{3}}{\sqrt{7+2\sqrt{12}}} = \frac{4+3\sqrt{3}}{2+\sqrt{3}}$$

$$= \frac{(4+3\sqrt{3})(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})}$$

$$= \frac{8-4\sqrt{3}+6\sqrt{3}-9}{4-3}$$

$= 2\sqrt{3}-1 = -1+\sqrt{12}$

37. Given $a^{\frac{1}{3}}+b^{\frac{1}{3}}+c^{\frac{1}{3}} = 0$

$\Rightarrow a^{\frac{1}{3}}+b^{\frac{1}{3}} = -c^{\frac{1}{3}}$

Taking cube on both sides

$$(a^{\frac{1}{3}}+b^{\frac{1}{3}})^3 = \left(-c^{\frac{1}{3}}\right)^3 = -c$$

$$a+b+3a^{\frac{1}{3}}b^{\frac{1}{3}}(a^{\frac{1}{3}}+b^{\frac{1}{3}}) = -c$$

$\therefore a+b+3a^{\frac{1}{3}}b^{\frac{1}{3}}(-c^{\frac{1}{3}}) = -c$

$\Rightarrow a+b+c = 3a^{\frac{1}{3}}b^{\frac{1}{3}}c^{\frac{1}{3}}$

$\Rightarrow (a+b+c)^3 = (3a^{\frac{1}{3}}b^{\frac{1}{3}}c^{\frac{1}{3}})^3 = 27\,abc$

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38. $\sqrt{16+6\sqrt{7}} = \sqrt{16+2\sqrt{63}} = \sqrt{16+2\sqrt{9\times 7}}$

$= \sqrt{9}+\sqrt{7} = 3+\sqrt{7}$

$\sqrt{16-6\sqrt{7}} = \sqrt{16-2\sqrt{63}} = \sqrt{16-2\sqrt{9\times 7}}$

$= \sqrt{9}-\sqrt{7} = 3-\sqrt{7}$

Given expression $= \dfrac{\sqrt{7}}{(3+\sqrt{7})-(3-\sqrt{7})} = \dfrac{7}{2\sqrt{7}} = \dfrac{1}{2}$

39. Given $3y + 2x \Rightarrow 3 = 2^{\frac{x}{y}} \Rightarrow 3^{\frac{1}{x}} = 2^{\frac{1}{y}}$ ...(i)

$12z = 2x \Rightarrow 12 = 2^{\frac{x}{z}} \Rightarrow 12^{\frac{1}{x}} = 2^{\frac{1}{z}}$ ...(ii)

Dividing $\dfrac{3^{\frac{1}{x}}}{12^{\frac{1}{x}}} = \dfrac{2^{\frac{1}{y}}}{2^{\frac{1}{z}}} = 2^{\frac{1}{y}-\frac{1}{z}}$

$\Rightarrow \dfrac{3^{\frac{1}{x}}}{3^{\frac{1}{x}}\times 4^{\frac{1}{x}}} = 2^{\frac{1}{y}-\frac{1}{z}} \Rightarrow 2^{-\frac{2}{x}} = 2^{\frac{1}{y}-\frac{1}{z}}$

$\Rightarrow \dfrac{1}{y}-\dfrac{1}{z} = -\dfrac{2}{x} \Rightarrow \dfrac{1}{z}-\dfrac{1}{y} = \dfrac{2}{x}$

40. Let $\dfrac{x}{b^2-c^2} = \dfrac{y}{c^2-a^2} = \dfrac{z}{a^2-b^2} = \lambda$

$x = \lambda(b^2 - c^2) \Rightarrow \dfrac{x}{b+c} = \lambda(b-c)$ ...(i)

$y = \lambda(c^2 - a^2) \Rightarrow \dfrac{y}{c+a} = \lambda(c-a)$ ...(ii)

$z = \lambda(a^2 - b^2) \Rightarrow \dfrac{z}{a+b} = \lambda(a-b)$ ...(iii)

Adding Eqs. (i), (ii), (iii)

$\dfrac{x}{b+c}+\dfrac{y}{c+a}+\dfrac{y}{a+b}$

$= \lambda[(b-c)+(c-a)+(a-b)]$

$= \lambda(0) = 0$

41. $2^a = 4^b = 8^c \Rightarrow 2^a = 2^{2b} = 2^{3c}$

$\Rightarrow a = 2^b = 3^c$

Since $abc = 288$

$\Rightarrow (3c)\left(\dfrac{3c}{2}\right)\times c = 288 \Rightarrow 9c^3 = 576$

$\Rightarrow c^3 = 64 \Rightarrow c = 4,\ a = 3c = 3\times 4 = 12$

and $b = \dfrac{a}{2} = \dfrac{12}{2} = 6$

$\therefore \dfrac{1}{2a}+\dfrac{1}{4b}+\dfrac{1}{6c} = \dfrac{1}{2\times 12}+\dfrac{1}{4\times 6}+\dfrac{1}{6\times 4}$

$= \dfrac{1}{24}+\dfrac{1}{24}+\dfrac{1}{24} = \dfrac{3}{24} = \dfrac{1}{8}$

42. $\dfrac{\sqrt{\frac{5}{2}}+\sqrt{7-3\sqrt{5}}}{\sqrt{\frac{7}{2}}+\sqrt{16-5\sqrt{7}}} = \dfrac{\frac{1}{\sqrt{2}}[\sqrt{5}+\sqrt{14-6\sqrt{5}}]}{\frac{1}{\sqrt{2}}[\sqrt{7}+\sqrt{32-10\sqrt{7}}]}$

$= \dfrac{[\sqrt{5}+\sqrt{14-2\sqrt{45}}}{[\sqrt{7}+\sqrt{32-2\sqrt{175}}}$

$= \dfrac{\sqrt{5}+\sqrt{(3-\sqrt{5})^2}}{\sqrt{7}+\sqrt{(5-\sqrt{7})^2}}$

$= \dfrac{\sqrt{5}+3-\sqrt{5}}{\sqrt{7}+5-\sqrt{7}} = \dfrac{3}{5}$

43. $x+\dfrac{1}{x} = \sqrt{3} \Rightarrow \left(x+\dfrac{1}{x}\right)^2 = 3$

$x^2+\dfrac{1}{x^2}+2 = 3 \Rightarrow x^2+\dfrac{1}{x^2} = 1$

44. $x+\dfrac{1}{x} = \sqrt{3}$

Taking cube $\left(x+\dfrac{1}{x}\right)^3 = x^3+\dfrac{1}{x^3}+3x\times\dfrac{1}{x}\left(x+\dfrac{1}{x}\right)$

$(\sqrt{3})^3 = x^3+\dfrac{1}{x^3}+3\sqrt{3}$

$\Rightarrow x^3+\dfrac{1}{x^3} = 0$

45. $xy = \dfrac{\sqrt{2}+1}{\sqrt{2}-1}\times\dfrac{\sqrt{2}-1}{\sqrt{2}+1} = 1$

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$\therefore \quad 3(xy + 1) = 3(1 + 1) = 6 \qquad ...(i)$

Now, $x + y = \dfrac{\sqrt{2}+1}{\sqrt{2}-1} \times \dfrac{\sqrt{2}-1}{\sqrt{2}+1} = \dfrac{(\sqrt{2}+1)^2 + (\sqrt{2}-1)^2}{(\sqrt{2}-1)(\sqrt{2}+1)}$

$= \dfrac{(2+1+2\sqrt{2})+(2+1-2\sqrt{2})}{(2-1)} = 6 \qquad ...(ii)$

$\therefore \quad 3(xy + 1) = x + y$

46. $y + \dfrac{1}{y} = \dfrac{\sqrt{x+a}-\sqrt{x-a}}{\sqrt{x+a}+\sqrt{x-a}} + \dfrac{\sqrt{x+a}+\sqrt{x-a}}{\sqrt{x+a}-\sqrt{x-a}}$

$= \dfrac{(\sqrt{x+a}-\sqrt{x-a})^2 + (\sqrt{x+a}+\sqrt{x-a})^2}{(\sqrt{x+a}+\sqrt{x-a})(\sqrt{x+a}-\sqrt{x-a})}$

$= \dfrac{4x}{2a} = \dfrac{2x}{a}$

47. $\dfrac{x+1}{x-1} - \dfrac{x-1}{x+1} = \dfrac{(x+1)^2(x-1)^2}{(x-1)(x+1)}$

$= \dfrac{(x^2+1+2x)-(x^2+1-2x)}{x^2-1}$

$= \dfrac{4x}{x^2-1} = \dfrac{4(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})^2-1}$

$= \dfrac{4(\sqrt{3}+\sqrt{2})}{(3+2+2\sqrt{6}-1)} = \dfrac{4(\sqrt{3}+\sqrt{2})}{4+2\sqrt{6}}$

$= \dfrac{4(\sqrt{3}+\sqrt{2})}{2\sqrt{2}(\sqrt{3}+\sqrt{2})} = \sqrt{2}$

48. Given expression

$\dfrac{(\sqrt{2}+1)^2}{3-\sqrt{2}} = \dfrac{2+1+2\sqrt{2}}{3-\sqrt{2}} = \dfrac{3+2\sqrt{2}}{3-\sqrt{2}} \times \dfrac{3+\sqrt{2}}{3+\sqrt{2}}$

$= \dfrac{9+3\sqrt{2}+6\sqrt{2}+4}{9-2}$

$= \dfrac{13+9\sqrt{2}}{7}$

$= \dfrac{13}{7} + \dfrac{9}{7}\sqrt{2} = a + b\sqrt{2}$

$\Rightarrow \quad a = \dfrac{13}{7}$ and $b = \dfrac{9}{7}$

$\Rightarrow \quad a + b = \dfrac{13}{7} + \dfrac{9}{7} = \dfrac{22}{7}$

49. Given expression

$\dfrac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}} + \dfrac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$

$= \dfrac{(1+\sqrt{2})(\sqrt{5}-\sqrt{3})+(1-\sqrt{2})(\sqrt{5}+\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}$

$= \dfrac{(\sqrt{5}-\sqrt{3}+\sqrt{10}-\sqrt{6})+(\sqrt{5}+\sqrt{3}-\sqrt{10}-\sqrt{6})}{(5-3)}$

$= \dfrac{2\sqrt{5}-2\sqrt{6}}{2} = \sqrt{5}-\sqrt{6}$

$\Rightarrow \quad \sqrt{5}-\sqrt{6} = a\sqrt{5}+b\sqrt{6} \Rightarrow a = 1, b = -1$

$\therefore \quad a + b = 0$

50. $\sqrt{1+a}+\sqrt{1-a} = \sqrt{1+\dfrac{\sqrt{3}}{2}} + \sqrt{1-\dfrac{\sqrt{3}}{2}}$

$\sqrt{1+\dfrac{\sqrt{3}}{2}} = \sqrt{\dfrac{2+\sqrt{3}}{2}} = \dfrac{\sqrt{4+2\sqrt{3}}}{4} = \dfrac{\sqrt{3}+1}{2}$

$\sqrt{1-\dfrac{\sqrt{3}}{2}} = \sqrt{\dfrac{2-\sqrt{3}}{2}} = \sqrt{\dfrac{4-2\sqrt{3}}{4}} = \dfrac{\sqrt{3}-1}{2}$

$\therefore \quad \sqrt{1+a}+\sqrt{1-a} = \dfrac{\sqrt{3}+1}{2} + \dfrac{\sqrt{3}-1}{2}$

$= \dfrac{2\sqrt{3}}{2} = \sqrt{3} = 2a$

51. Given $x = a^p$ and $y = a^q$

Now, $x^q y^p = (a^p)^q (a^q)^p = a^{\frac{2}{r}}$

$\Rightarrow \quad a^{pq} \times a^{pq} = a^{\frac{2}{r}}$

$\Rightarrow \quad 2pq = \dfrac{2}{r}$

$\Rightarrow \quad pqr = 1$

52. Given expression

$\dfrac{1}{1+x^{a-b}} + \dfrac{1}{1+x^{b-a}} = \dfrac{x^{-a}}{x^{-a}+x^{-b}} + \dfrac{x^{-b}}{x^{-b}+x^{-a}}$

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$$= \frac{x^{-a}+x^{-b}}{x^{-a}+x^{-b}} = 1$$

53. $x = \frac{2}{3+\sqrt{7}} \times \frac{3-\sqrt{7}}{3-\sqrt{7}} = \frac{2(3-\sqrt{7})}{9-7} = 3-\sqrt{7}$

$x - 3 = -\sqrt{7}$

$\Rightarrow \quad (x-3)^2 = 7$

54. Let $\sqrt{8+2\sqrt{15}} = \sqrt{a}+\sqrt{b}$

$\therefore \quad 8+2\sqrt{15} = a+b+2\sqrt{ab}$

$\Rightarrow \quad a + b = 8$

and $\quad ab = 15$

$\Rightarrow \quad a = 5, b = 3$

$\therefore \quad \sqrt{8+2\sqrt{15}} = \sqrt{5}+\sqrt{3}$

55. $x+\frac{1}{x} = 2\sqrt{2}+\sqrt{7}+\frac{1}{2\sqrt{2}+\sqrt{7}}$

$= 2\sqrt{2}+\sqrt{7}+\frac{1}{(2\sqrt{2}+\sqrt{7})} \times \frac{(2\sqrt{2}-\sqrt{7})}{(2\sqrt{2}-\sqrt{7})}$

$= 2\sqrt{2}+\sqrt{7}+\frac{(2\sqrt{2}-\sqrt{7})}{(8-7)}$

$= 2\sqrt{2}+2\sqrt{2} = 4\sqrt{2}$

56. Let $x = \sqrt{2+\sqrt{2+\sqrt{2+\ldots\infty}}} = \sqrt{2+x}$

On squaring, $x = 2 + x$

$\Rightarrow \quad x^2 - x - 2 = 0$

$\Rightarrow \quad (x-2)(x+1) = 0$

$\Rightarrow \quad x = 2$ or -1

$x = -1$ is not possible, hence $x = 2$

57. $\frac{1}{a} = \frac{1}{\sqrt{7}-\sqrt{5}} \times \frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}+\sqrt{5}}$

$\Rightarrow \quad \frac{1}{a} = \frac{\sqrt{7}+\sqrt{5}}{2}$

$\frac{1}{b} = \frac{1}{\sqrt{13}-\sqrt{11}} \times \frac{\sqrt{13}+\sqrt{11}}{\sqrt{13}+\sqrt{11}}$

$\Rightarrow \quad \frac{1}{b} = \frac{\sqrt{13}+\sqrt{11}}{2}$

$\therefore \quad \frac{1}{b} > \frac{1}{a}$ □ a > b

58. (b) $8^{\frac{2}{3}} - 16^{\frac{1}{4}} + 9^{\frac{1}{2}} = 4 - 2 + 3 = 5$

59. $9^{2x-1} = 27^{x+2}$

$34x - 2 = 33x + 6 \Rightarrow 4x - 2 = 3x + 6 \Rightarrow x = 8$

60. $x^{\left(\frac{a^5}{3a^{2bc}}\right)} x^{\left(\frac{b^5}{3ab^3c}\right)} x^{\left(\frac{c^5}{3abc^3}\right)}$

$= \frac{a^3}{x^3abc} + \frac{b^3}{3abc} + \frac{c^3}{3abc} = x^{\frac{a^3+b^3+c^3}{3abc}}$

$= x^{\frac{3abc}{3abc}} = \left[\begin{array}{l} \because a+b+c=0 \\ \therefore a^3+b^3+c^3 = 3abc \end{array}\right]$

61. 3^{33333} is the greatest number.

62. $\left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}} = 2^x \Rightarrow \sqrt{2}^{\sqrt{2}\times\sqrt{2}} = 2^x$

$\Rightarrow \quad \sqrt{2^2} = 2^x \Rightarrow 2 = 2^x$

$\Rightarrow \quad x = 1$

63. $\frac{5^{23}\times\sqrt[3]{5^8}}{\sqrt[3]{5^7}} = \frac{5^{23}\times 5^{8/3}}{5^{7/3}} = 5^{\frac{2}{3}+\frac{8}{3}-\frac{7}{3}} = 5$

64. $\frac{5\times(2^{k-2})+10\times(2^{k-1})}{10^{k+2}} = \frac{2^5[5\times 2^{-2}+10\times 2^{-1}]}{2^{k+2}\times 5^{k+2}}$

$= \frac{25/4}{4\times 5^{k+2}} = \frac{1}{16\times 5^k}$

65. $\left(\frac{\sqrt[4]{ab}-\sqrt{b}}{\sqrt{a}-\sqrt[4]{ab}}\right)^{-4} = \left[\frac{b^{1/4}[a^{1/4}-b^{1/4}}{a^{1/4}[a^{1/4}-b^{1/4}]}\right]^{-4} = \left[\frac{b^{1/4}}{a^{1/4}}\right]^{-4} = \frac{a}{b}$

66. $3^{x+1} + 3^{2x+1} = 270$

Let $\quad 3^x = t$

$3t + 3t^2 = 270$

$t^2 + 10t - 90 = 0$

$(t+10)(t-9) = 0$

$t = 9 = 3x$

$3^2 = 3^x$

$x = 2$

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67. $5^{x-2} \times 2^{3x/(x+1)} = 4$

$5^{x-2} \times 2^{3x/(x+1)} = 5^0 \times 2^2$

Equation the indicates of equal bases,

$x - 2 = 0$ and $3x/(x + 1) = 2$

From both these questions

$x = 2$

68. Let $3^x + 3^{-x} = a$

Let $3x = K$

Then, $a = K + 1/K = (K^2 + 1)/K$

ie, $K^2 - Ka + 1 = 0$

For this to have real roots, the discriminant should be greater than or equal to 0.

ie, $a^2 - 4 \geq 0$

or $a \geq /2/2a \geq 2$

or $a < -2$

(not possible)

Least value = 2.

69. $t = 2 + \sqrt[3]{4} + \sqrt[3]{2}$

$t - 2 = \sqrt[3]{4} + \sqrt[3]{2}$

$= 2^{2/3} + 2^{1/3} = 2^{1/3}(1 + 2^{1/3})$

$(t - 2)^3 = [2^{1/3}(1 + 2^{1/3})]^3 = 2(1 + 2^{1/3})^3$

$= 2[1 + 2 + 3 \times 2^{1/3}(1 + 2^{1/3})]$

70. $a = -5$ and $b = -6$

$(a - b)^{b-a} + (b - a)^{a-b}$

$= (-5 + 6)^{-6+5} + (-6 + 5)^{-5+6}$

$= (1)^{-1} + (-1)^1 = 1 - 1 = 0$

71. Remaining NCC cadets = 6440 – 40 = 6400

So, NCC cadets in each row = $\sqrt{6400} = 80$

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