CHAPTER 21

Clocks and Calendar

Clock

A clock has 2 hands, the smaller one is called the hour hand or short hand while the larger one is called the minute hand or long hand.

The face of a clock is a circle which subtends an angle of 360° at the centre.

Some Important Points

In every hour

1. (a) Both the I ands coincide once. At this point the angle between them is 0°..
   (b) The hands are straight (point in opposite directions) once. At this point the angles between them are 180°.
   (c) The hands are twice perpendicular to each other. At this point the angle between them is 90°.
2. (a) In 60 min the minute hand covers 360°.

   Thus, in 1 min the minute hand covers $\left(\frac{360}{60}\right)^{\circ} = 6^{\circ}$

   (b) In 12 h the hour hand covers 360°.

   Thus, in 1 min the hour hand covers $\left(\frac{360}{12\times 60}\right)^{\circ} = \frac{1^{\circ}}{2}$

   Thus, in of to minute, the minute hand gains $\left(6-\frac{1}{2}\right)^{\circ} = 5\frac{1^{\circ}}{2}$, than the hour hand.
3. (a) When the two hands are at right angles, they are 15min spaces apart.
   (b) When the two hands are in opposite directions, they are 30 min spaces apart.
   (c) In 60 min the minute hand gains 55 min on the hour hand.
   (d) The minute hand moves 12 times as fast as the hour hand.
4. (a) **Too Fast:** If a clock indicates 7 : 1.5, when the correct time is 7, it is said to be 15 min too fast.
   (b) **Too Slow:** If a clock indicates 7 : 30, when the correct time is 7 45 , it is said to be 15 min too slow.

Calendar

1. **Odd Days:** The number of days more than the complete weeks for a given period called odd days.
2. **Ordinary Year:** An ordinary year has 365 days and an ordinary year is not a leap year.
3. **Leap Year:**
   (a) Any year (except a century) which is divisible by 4 is a leap year.
   (b) However, every 4th century is a leap year, ie, a century is a leap year when it is divisible by 400.
   (c) A leap year has 366 days.

   Examples of Leap Year 1924, 1908, 1944, 2008, 1684, etc., are all leap years.
   400, 800, 1200, 1600, 2000, etc., are all leap years.
4. **Counting of Odd Days**
   (a) 1 ordinary year = 365 days = (52 weeks + 1 day) $\Rightarrow$ 1 ordinary year = 1 odd day
   (b) 1 leap year = 366 days = (52 weeks + 2 days) = 1 leap year = 2 odd days
   (c) 100 yr = 76 ordinary years + 24 leap years = (76 × 1 + 24 × 2) odd days = 124 odd days
   = (17 weeks + 5 days) = 5 odd days
   (d) 200 yr = (5 × 2) = 10 odd days = (1 week + 3 days) = 3 odd day
   (e) 300 yr = (5 × 3) = 15 odd days = (2 weeks + 1 day) = 1 odd days
   (f) 400 yr = (5 × 4 + 1) = 21 odd days = (3 weeks + 0 day) = 0 odd .lay Similarly, 800 yr, 1200 yr, 1600 yr, 2000 yr have 0 odd days.
5. Day of the week with respect to the number of odd days.

Number of odd days	Day of the week
0	Sunday
1	Monday
2	Tuesday
3	Wednesday
4	Thursday
5	Friday
6	Saturday

Example 1: Find the angle between the hour hand and the minute hand of a clock when the time is 5 : 35.

Solution. Angle traced by hour hand in 12 h = 360°

Angle traced by the hour hand in $5\frac{35}{60}\text{h} = \frac{67}{12}\text{h} = \left(\frac{360}{12}\times\frac{67}{12}\right)^\circ = \left(167\frac{1}{2}\right)^\circ$

Angle traced by minute hand in 60 min = 360°

Angle traced by minute hand in 35 min = $\left(\frac{360}{60}\times 35\right)^\circ = 210°$

Required Angle = $\left(210° - 167\frac{1}{2}^\circ\right) = 42\frac{1}{2}^\circ$

Example 2: At what time between 8 and 9 O'clock will the hands of a clock be in the same straight line but not together ?

Solution. At 8 O'clock the hour hand is at 8 and the minute hand is at 12. Thus, the two hands are 20 min spaces apart.

To be in the same straight line but not together, they will be 30 min spaces apart. .% he minute hand will gain (30 – 20) min = 10 min spaces over the hour hand. 55 min spaces are gained by hour hand in 60 min.

10 min spaces will be gained by hour hand in 60 × 10 min = $10\frac{10}{11}$ min

∴ The hands will be in the same straight line but not together at $10\frac{10}{11}$ min pest 8.

Example 3: If the hands of a clock coincide every 65 min (true time) in 24 h. How much a day does the clock gain ?

Solution. 55 rain are gained by mintue hand in 60 min.

60 min will be gained by minute hand in $\left(\frac{60}{55}\times 60\right)$min = $\frac{720}{11}$ min = $65\frac{5}{11}$ min

∴ The hands of a correct clock coincide every $65\frac{5}{11}$ min.

But the hands of the clock in question coincide every 65 min.

∴ In every 65 min the clock in question gains $\frac{5}{11}$ min

∴ In 24 h the clock in question gains = $\left(\frac{5}{11}\times\frac{1}{65}\times 24\times 60\right)$ min = $\frac{1440}{143}$ min = $10\frac{10}{143}$ min.

Example 4: A watch which gains uniformly is 4 min slow at 7 pm on Mo day and is 4 min 48 s fast at 7 pm on following Monday. When was the watch correct ?

Solution. Total time in hours from Monday at 7 pm to the following Monday at 7 pm = (7 × 241, *h* = 168h.

In 168 h the watch gains $\left(4+4\frac{2}{5}\right)$ min.

$$= \left(4+\frac{22}{5}\right) \text{min} = \left(\frac{22}{5}\right) \text{min}$$

Now, $\frac{42}{5}$ min are gained in 168 h.

∴ 4 min are gained in $\left(168\times\frac{5}{42}\times 4\right)$h = 80 h = 3 days and 8 h

∴Watch is correct 3 days and 8 h after 7 pin Mondav ie, it will be correct at 3 am on Friday.

Example 5: A clock gains 10 min in every 24 h. It is set , ht on Tuesday at 9 am. Whet will be the correct time on the following Thursday, when the watch indicates 7 pm.

Solution. Time from Tuesday at 9 am to the following Thursday at 7 pm = (24 × 2 + 10) h = 58h 24 h 10 min of the clock in question = 24 h of the correct clock h of the clock in question = 24 h of the correct clock 40 h of the clock in question = $\left(24\times\frac{6}{145}\times 58\right)$h of the correct clock = 57 h 36 min of correct clock

∴ The correct time is 57 h 36 ml after 9 am on Tuesday

Thus, the correct time on the following Wednesday will be 6 : 36 pm.

Example 6: What was the day of the week on 26th June, 1816 ?

Solution. 26th June 1816 = (1815 yr + Period from 1.1.1816 to 26.6.1816) counting of odd days

Number of odd days in 1600 yr = 0

Number of odd days in 200 yr = 3

15 yr = 3 leap years + 12 ordinary years = (3 × 2 + 12 × 1) odd days = 18 odd days

(2 weeks + 4 days) = 4 odd days

1815 have = (0 + 3 + 4) = 7 odd days = 0 odd day

Jan Feb March April May June

(31 + 29 + 31 + 30 + 31 + 261=178 days 178 days = (25 weeks + 3 days) = 3 odd days

Total number of odd days = (0 + 3) = 3 odd days Hence, the required Jay is Wednesday.