

UPSC 2010 Prelims
MATHEMATICS

Question Paper
And
Answer Keys
Booklet Series D

Solved By: IMS Institute of Mathematical
Sciences

Compiled by: www.upscportal.com

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T.B.C. : N-DTQ-K-OCV

TEST BOOKLET SERIES

Serial No

**TEST BOOKLET
MATHEMATICS**

D

Time Allowed : Two Hours

Maximum Marks : 300

INSTRUCTIONS

1. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS TEST BOOKLET DOES **NOT** HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS, ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.
2. ENCODE CLEARLY THE TEST BOOKLET SERIES A, B, C OR D AS THE CASE MAY BE IN THE APPROPRIATE PLACE IN THE ANSWER SHEET.
3. You have to enter your Roll Number on the Test Booklet in the Box provided alongside. **DO NOT** write *anything else* on the Test Booklet.
4. This Test Booklet contains **100** items (questions). Each item is printed both in **Hindi and English**. Each item comprises four responses (answers). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose **ONLY ONE** response for each item.
5. You have to mark all your responses **ONLY** on the separate Answer Sheet provided. See directions in the Answer Sheet.
6. All items carry equal marks.
7. Before you proceed to mark in the Answer Sheet the response to various items in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per instructions sent to you with your Admission Certificate.
8. After you have completed filling in all your responses on the Answer Sheet and the examination has concluded, you should hand over to the Invigilator **only the Answer Sheet**. You are permitted to take away with you the Test Booklet.
9. Sheets for rough work are appended in the Test Booklet at the end.
10. **Penalty for wrong answers :**
THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.
 - (i) There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third (0.33)** of the marks assigned to that question will be deducted as penalty.
 - (ii) If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that question.
 - (iii) If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.

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ध्यान दें : अनुदेशों का हिन्दी रूपान्तर इस पुस्तिका के पिछले पृष्ठ पर छापा है।

40 x 2
= 80
20
100

1. What is the dimension of a space W generated by $\{(1, -4, -2, 1), (1, -3, -1, 2), (3, -8, -2, 7)\}$?

- (a) 1
- (b) 2
- (c) 3
- (d) 4

2. Consider the following statements associated with an n -dimensional vector space V :

- 1. Every basis of V contains exactly n vectors.
- 2. No linearly independent subset of V has more than n vectors.

Which of the statements given above is/are correct?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

3. Let $T: R^n \rightarrow R^n$ and $T(x_0) = 0$ for some $x_0 \neq 0, x_0 \in R^n$. Then there exists a matrix A of order n such that

- (a) $Tx = Ax$ and A is non-singular
- (b) $Tx = Ax$ and A is singular
- (c) $\text{Ker } T = \{0\} = N(A)$
- (d) None of the above

4. Consider the following statements:

1. The number of diagonal matrices

$$M = \begin{pmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{pmatrix} \in M_3(\mathbb{R}) \text{ where } \mathbb{R}$$

is the set of real numbers such that $M^5 = M$, is 27.

2. The number of diagonal matrices

$$M = \begin{pmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{pmatrix} \in M_3(\mathbb{R}) \text{ where } \mathbb{R}$$

is the set of real numbers such that $M^4 = M$, is 9.

Which of the statements given above is/are correct?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

5. Let $A = \begin{pmatrix} 4 & 5 \\ 3 & 4 \\ 2 & 3 \end{pmatrix} \in M_{3 \times 2}(\mathbb{Z})$ be a 3×2 matrix

over the set of integers.

Consider the following statements:

1. For any $n \in \mathbb{Z}$, there exists a unique $(x, y) \in \mathbb{Z} \times \mathbb{Z}$ such that $4x + 3y + 2n = 1$ and $5x + 4y + 2n = 0$.

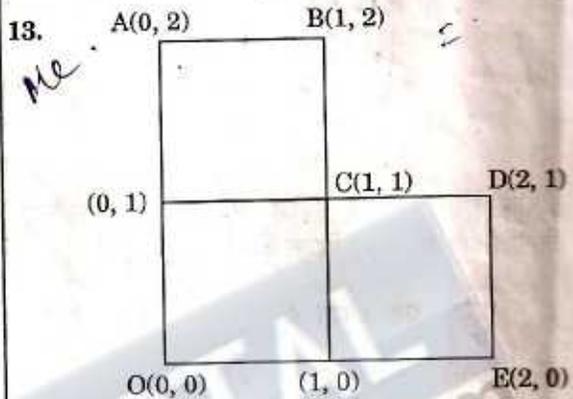
2. There exist only finitely many $B \in M_{2 \times 3}(\mathbb{Z})$ such that $BA = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$.

Which of the statements given above is/are correct?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

6. What is the number of tangents to the curve $y = x^2$ passing through the point $(1, -3)$?
 (a) 0
 (b) 1
 (c) 2
 (d) 3
7. Let $f(x) = (x + 1)^2$ for $x \geq -1$. If $g(x)$ is the function whose graph is the reflection of $f(x)$ with respect to $y = x$, then what does $g(x)$ equal to ?
 (a) $-\sqrt{x} - 1, x \geq 0$
 (b) $(x + 1)^{-2}, x > -1$
 (c) $\sqrt{x} + 1, x \geq -1$
 (d) $\sqrt{x} - 1, x \geq 0$
8. What is the curvature of the curve $y = x^4 - 4x^3 + 20x^2$ at the origin ?
 (a) $1/10$
 (b) $1/40$
 (c) 10
 (d) 40
9. The asteroid $x = \sqrt{5} \cos^3 t, y = \sqrt{5} \sin^3 t$ is revolved about x-axis. What is the surface area of the solid of revolution ?
 (a) 12π square unit
 (b) 10π square unit
 (c) 8π square unit
 (d) 6π square unit
10. If the line $y = x$ is tangential to the parabola $y = x^2 + bx + c$ at the point $(1, 1)$, what are the values of b, c respectively ?
 (a) $0, -1$
 (b) $1, 1$
 (c) $-1, -1$
 (d) $-1, 1$
11. Let the equation of the curve be $x = a(\theta + \sin \theta), y = a(1 - \cos \theta)$. If θ changes at a constant rate k , then what is the rate of change of the slope of the tangent to the curve at $\theta = \pi/3$?
 (a) k
 (b) $k/2$
 (c) $k/3$
 (d) $2k/3$

12. If $x + 4|y| = 6y$, then y as a function of x is
 (a) continuous at $x = 0$
 (b) derivable at $x = 0$
 (c) $\frac{dy}{dx} = \frac{1}{2}$ for all x
 (d) None of the above



Where is the centre of mass of the lamina OABCDE (shown above) of 30 g ?

- (a) $(1, 1)$
 (b) $(3/2, 3/2)$
 (c) $(5/6, 5/6)$
 (d) None of the above
14. A uniform triangular lamina ABC is suspended from the corner A and in equilibrium the side BC makes an angle α with the horizontal. Which one of the following is correct ?
 (a) $2 \tan \alpha = \cot B - \cot C$
 (b) $\tan \alpha = \cot B - \cot C$
 (c) $2 \cot \alpha = \tan B - \tan C$
 (d) $\cot \alpha = \tan B - \tan C$
15. If the gravitational force varies inversely as the n^{th} power of the distance, then the time period of a satellite revolving round the earth in a circular orbit of radius r is proportional to
 (a) $r^{(n+1)/2}$
 (b) $r^{(n-1)/2}$
 (c) $r^{-(n+1)/2}$
 (d) $r^{-(n-1)/2}$

Directions : For the three (03) items which follow :

Consider the curve $x(x^2 + y^2) = a(x^2 - y^2)$

16. What is the area of the loop of the curve ?

(a) $\left(2 - \frac{\pi}{2}\right)a^2$

(b) $\left(2 + \frac{\pi}{2}\right)a^2$

(c) $\left(1 - \frac{\pi}{4}\right)a^2$

(d) None of the above

17. What is area between the curve and the line $x + a = 0$?

(a) $\left(2 - \frac{\pi}{2}\right)a^2$

(b) $\left(2 + \frac{\pi}{2}\right)a^2$

(c) $\left(1 + \frac{\pi}{4}\right)a^2$

(d) None of the above

18. Consider the following statements :

1. There are two asymptotes of the curve parallel to coordinate axes.
2. There are no oblique asymptotes of the curve.

Which of the above statements is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

Directions : The following four (04) items consist of two statements, Statement I and Statement II. You are to examine these two statements carefully and select the answers to these items using the code given below :

Code :

(a) Both the statements are individually true and Statement II is the correct explanation of Statement I.

(b) Both the statements are individually true but Statement II is *not* the correct explanation of Statement I.

(c) Statement I is true but Statement II is false.

(d) Statement I is false but Statement II is true.

19. **Statement I :** Let H be a subgroup of $(\mathbb{Q}, +)$ where Q is the set of rationals and $H \neq \mathbb{Q}$. Then Q is not a subset of $\left\{\frac{1}{n}h : h \in H\right\}$ for any positive integer n.

Statement II : If G is any group and H is a subgroup of G, $H \neq G$, then $\{g^n : g \in G\}$ is not a subset of H for any positive integer n.

20. Let $f : \mathbb{R} \rightarrow M_3(\mathbb{R})$ where $M_3(\mathbb{R})$ is the set of 3×3 matrices over \mathbb{R} , be defined as

$$f(x) = \begin{pmatrix} 1 & 1+x & x \\ 2 & 1-x & x \\ 3 & x & 4 \end{pmatrix}$$

Statement I : There exist infinite number of $x \in \mathbb{R}$ for which determinant of $f(x)$ is positive.

Statement II : There are exactly three real numbers x such that determinant of $f(x) = 0$.

21. Consider $\mathbb{R}^* = \mathbb{R} \setminus \{0\}$ which is a group under multiplication.

Let $f: \mathbb{R}^* \rightarrow \mathbb{R}^*$ be given by

$$f(r) = 1 \text{ if } r > 0 \\ = -1 \text{ if } r < 0$$

Statement I: For any $r_1, r_2 \in \mathbb{R}^*$, $f(r_1 r_2) = f(r_1) f(r_2)$.

Statement II: $H = \{r \in \mathbb{R}^* : f(r) = 1\}$ is a subgroup of \mathbb{R}^* and $\mathbb{R}^* = H \cup \{-r : r \in H\}$.

22. Statement I: The lines $ax + by + c = 0$, $bx + cy + a = 0$, $cx + ay + b = 0$ are concurrent if $a^3 + b^3 + c^3 = 3abc$.

Statement II: The above three lines are concurrent because the product of their slopes is -1 .

23. A particle is projected with a velocity of 10 m/s at an angle of elevation of 60° with the horizontal. What is the equation to trajectory of the projectile ($g = 10 \text{ ms}^{-2}$)?

- (a) $2y = 2\sqrt{3}x - x^2$
- (b) $2y = \sqrt{3}x - x^2$
- (c) $y = 2\sqrt{3}x - x^2$
- (d) None of the above

24. A particle executes simple harmonic motion along a straight line given by $x = a \cos(nt + \theta)$ where a, n, θ are constants; x is the distance of the particle from a fixed point O on the line at time t . The velocity of the particle will be the least in magnitude when its distance from the fixed point is

- (a) Zero
- (b) $\pm a$
- (c) $\pm a/2$
- (d) $\pm a/4$

25. The law of motion in a straight line is given by $s = vt/2$, where v is the velocity at any instant t . Then which one of the following is correct about the acceleration f ?

- (a) f is constant
- (b) f must be zero
- (c) f is directly proportional to v
- (d) f is inversely proportional to v

26. A particle of unit mass, initially at rest at a distance R from a fixed point O is acted upon by a centre of repulsion situated at O repelling with a force k/x^2 , where k is a constant and v is the velocity of the particle at a distance x from O at time t . Which one of the following is correct?

- (a) $Rxv^2 = 2k(R - x)$
- (b) $Rxv^2 = 2k(R - x)^2$
- (c) $Rxv = 2k(R - x)$
- (d) $Rxv^2 = 2k(x - R)$

27. The component of a vector $\vec{\mu}$ perpendicular to the vector $\vec{\lambda}$ is $\vec{\mu} - \vec{v}$. What is \vec{v} equal to?

- (a) $\frac{\vec{\lambda} \cdot \vec{\mu}}{|\lambda|} \vec{\mu}$
- (b) $\frac{\vec{\lambda} \cdot \vec{\mu}}{|\lambda|^2} \vec{\lambda}$
- (c) $\frac{\vec{\lambda} \cdot \vec{\mu}}{|\lambda|^2} \vec{\mu}$
- (d) None of the above

28. What is the area bounded by the line $x \cos \alpha + y \sin \alpha = p$, the x-axis and the normal to the line from origin (α is the angle the normal from origin makes with the x-axis)?

- (a) $(p^2 \cos^2 \alpha)/2$
- (b) $p^2 \cos \alpha$
- (c) $p^2 \tan \alpha$
- (d) $(p^2 \tan \alpha)/2$

29. The equation to circle in polar coordinates is given by $r = a \cos \theta + b \sin \theta$. What is the centre of the circle?

- (a) $(a^2 + b^2)^{1/2}, \tan^{-1}\left(\frac{b}{a}\right)$
- (b) $(a^2 + b^2)^{1/2}, \tan^{-1}\left(\frac{a}{b}\right)$
- (c) $\frac{(a^2 + b^2)^{1/2}}{2}, \tan^{-1}\left(\frac{b}{a}\right)$
- (d) $\frac{(a^2 + b^2)^{1/2}}{2}, \tan^{-1}\left(\frac{a}{b}\right)$

Handwritten notes:
 $ax + by = a^2 + b^2$
 $x \cos \theta + y \sin \theta = a \cos \theta + b \sin \theta$
 (a, b)
 $(\cos \theta, \sin \theta)$
 $r = a \cos \theta + b \sin \theta$

30. OABC is a unit square where O is the origin. The side OA makes an angle 45° with the x-axis. What is the equation of the diagonal AC? (The square lies in first and fourth quadrants)

- (a) $y = \frac{1}{\sqrt{2}}$
- (b) $y = \frac{1}{2}$
- (c) $y = 1$
- (d) $x = \frac{1}{\sqrt{2}}$

Handwritten notes:
 (r, θ)
 $r =$

31. The curve in which the gradient of the normal at any point equals the ratio of ordinate to the abscissa at that point is

- (a) an ellipse
- (b) a hyperbola
- (c) a parabola
- (d) a circle

Handwritten notes:
 $\frac{y}{x}$
 $y = mx$
 $\frac{dy}{dx} = m$

32. What is the volume generated by $y = a \sin(\pi x)$ as the area lying between $x = 0$ and $x = 2$ revolves about x-axis?

- (a) πa^2
- (b) πa^3
- (c) $4\pi a^3/3$
- (d) $2\pi a^3/3$

33. What is $(\vec{a} \times \vec{b}) \times (\vec{a} \times \vec{c}) \cdot \vec{d}$ equal to?

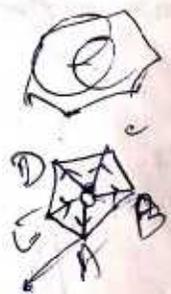
- (a) $\vec{d} \times \vec{a} [\vec{a} \vec{b} \vec{c}]$
- (b) $-\vec{d} \times \vec{a} [\vec{a} \vec{b} \vec{c}]$
- (c) $\vec{d} \cdot \vec{b} [\vec{a} \vec{b} \vec{c}]$
- (d) $\vec{a} \cdot \vec{d} [\vec{a} \vec{b} \vec{c}]$

34. A, B, C, D, E are the vertices of a regular pentagon whose centre is O such that

$$\vec{OA} + \vec{OB} + \vec{OC} + \vec{OD} = \lambda \vec{EO}$$

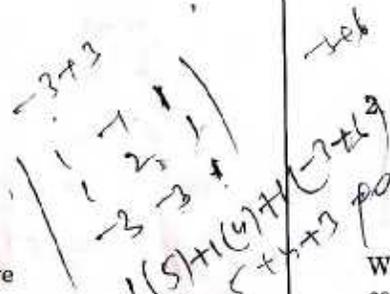
What is the value of λ ?

- (a) 0
- (b) 1
- (c) -1
- (d) -4



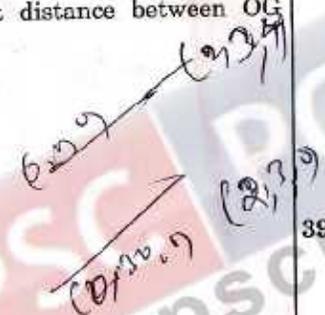
35. Let $\vec{a} = \hat{i} - \hat{j}$ and $\vec{b} = \hat{i} + 2\hat{j}$. What is the vector coplanar with \vec{a} , \vec{b} and perpendicular to \vec{a} ?

- (a) $2\hat{i} + \hat{j}$
- (b) $\hat{i} + 2\hat{j}$
- (c) $-3\hat{i} - 3\hat{j}$
- (d) None of the above



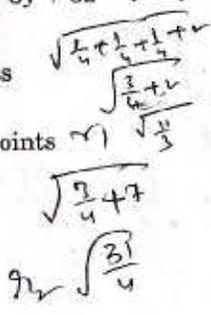
36. The vertices of a rectangular parallelepiped are $O(0, 0, 0)$, $A(2, 0, 0)$, $B(0, 3, 0)$, $C(0, 0, 4)$, $D(2, 3, 0)$, $E(0, 3, 4)$, $F(2, 0, 4)$ and $G(2, 3, 4)$. What is the shortest distance between OG and BD ?

- (a) $\frac{6}{\sqrt{13}}$
- (b) $\frac{12}{5}$
- (c) $\frac{4}{\sqrt{5}}$
- (d) None of the above



37. The spheres $x^2 + y^2 + z^2 + x + y + z - 2 = 0$ and $3x^2 + 3y^2 + 3z^2 + 3x + 3y + 3z - 7 = 0$

- (a) intersect in two points
- (b) intersect in infinite points
- (c) touch each other
- (d) do not intersect



38. Consider the following statements :

1. The set of planes $2x + y + z + 4 = 0$, $y - z + 4 = 0$, $3x + 2y + z + 8 = 0$ intersect in a line.

The set of planes $x - z - 1 = 0$, $x + y - 2z - 3 = 0$, $x - 2y + z - 3 = 0$ form a triangular prism.

Which of the statements given above is/are correct?

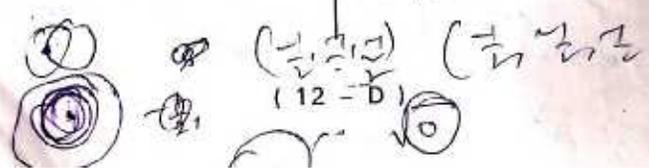
- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

39. A cylinder is a surface generated by a straight line which is always parallel to a fixed line and is subjected to which of the following conditions?

- 1. It may intersect a given curve.
- 2. It may touch a given surface.

Which of the conditions given above is/are correct?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Either 1 or 2



40. Six forces, each equal to P, act along the edges of a cube of side length 'a' taken in order, which do not meet a given diagonal. Their resultant is a couple of magnitude

(a) 2Pa
 (b) $2\sqrt{3}Pa$
 (c) $3\sqrt{2}Pa$
 (d) None of the above

41. If the forces 8P and 6P acting at a point (3, 2) in Cartesian rectangular coordinate plane are parallel to the positive direction of x and y axes respectively, then what is the algebraic sum of the moments of the forces about the origin?

(a) 2P
 (b) 8P
 (c) 14P
 (d) -2P

42. Forces of magnitudes 2 N, $\sqrt{3}$ N, 5 N, $\sqrt{3}$ N and 2 N act respectively at one of the angular points of a regular hexagon directed towards the five other angular points. Then what is the magnitude of the resultant?

(a) 5 N
 (b) 10 N
 (c) 15 N
 (d) 20 N

43. Let \mathbb{R} denote the set of all real numbers. Let $A_n = \{x \in \mathbb{R} : x^{2^n} \text{ is a rational number}\}$ for $n = 1, 2, 3, \dots$. Let $B = \{x \in \mathbb{R} : x^3 \text{ is a rational number}\}$. Consider the following statements:

1. $A_n \cap B$ is a proper subset of $A_{n+1} \cap B$ for each $n \geq 1$.
 2. A_n is a proper subset of A_{n+1} for each $n \geq 1$.
 Which of the statements given above is/are correct?

(a) 1 only
 (b) 2 only
 (c) Both 1 and 2
 (d) Neither 1 nor 2

44. Consider the following statements:

1. Let $A = \{1, 2, 3, 4, 5, 6\}$. Under multiplication modulo 7 there exist at least three elements $a \in A$ such that $a^2 = 1$.
 2. Let G be a finite group. Let $S = \{x \in G : x^3 = e, x \neq e\}$. Then the number of elements in G is even.
 Which of the statements given above is/are correct?

(a) 1 only
 (b) 2 only
 (c) Both 1 and 2
 (d) Neither 1 nor 2

45. If $V_1 = \{(a, b, c, d) : b - 2c + d = 0\}$, $V_2 = \{(a, b, c, d) : a = d, b = 2c\}$, what is the basis of $V_1 \cap V_2$?

(a) $\{(0, -1, 0, 1), (0, 2, 0, 1), (1, 0, 0, 0)\}$
 (b) $\{(0, 2, 1, 0)\}$
 (c) $\{(0, 2, 1, 0), (1, 0, 0, -1)\}$
 (d) $\{(1, 0, 0, -1)\}$

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(14-D) (a, b, c, d)
 $(a, b, c, 2c-b)$
 $(1, 0, 0, 0)$
 $(0, 2, 1, 0)$

46. If one of the two forces acting on a particle is double that of the other and if θ is the angle between the directions of the resultant and the greater force, then which one of the following is correct?

- (a) θ cannot be greater than $\pi/6$
- (b) $\theta > \pi/6$
- (c) $\theta < \pi/6$
- (d) None of the above

47. For the curve $x^3 + y^3 = 3x^2$, what is the oblique asymptote?

- (a) $x + y = 0$
- (b) $x + y = 1$
- (c) $x - y = 0$
- (d) None of the above

$1+m^3=0$
 $(1+m)(1-m+m^2)=0$
 $m=1$
 $\phi_2 = 3$
 $\phi_2(m)$
 $\phi_3'(m)$

48. What is the kernel of $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} x+z \\ x+y+2z \\ 2x+y+3z \end{pmatrix}$?

- (a) $\{\alpha(1, 1, -1) \mid \alpha \in \mathbb{R}\}$
- (b) $\{\alpha(1, 1, 0) \mid \alpha \in \mathbb{R}\}$
- (c) $\{\alpha(1, 0, -1) \mid \alpha \in \mathbb{R}\}$
- (d) None of the above

$= \frac{z}{3m^2}$
 $\frac{z}{3} = 1$
 $y = 2m + \alpha$
 $z = -\alpha + 1$
 $x + y = 1$

49. What is the value of the parameter α for which the function $f(x) = 1 + \alpha x$, $\alpha \neq 0$ is the inverse of itself?

- (a) -2
- (b) -1
- (c) 1
- (d) 2

$y = 1 + \alpha x$
 $x = 1 + \alpha y$

50. What is the equation of the plane which passes through the z-axis and is perpendicular to the line $\frac{x-1}{\cos \theta} = \frac{y+2}{\sin \theta} = \frac{z-3}{0}$?

- (a) $x + y \tan \theta = 0$
- (b) $x \tan \theta + y = 0$
- (c) $x - y \tan \theta = 0$
- (d) $x \tan \theta - y = 0$

51. What is the set of all points where the second derivative of the function $x|x|$ exists?

- (a) The set of all real numbers
- (b) The set of all non-zero real numbers
- (c) The set of all positive real numbers only
- (d) The set of all negative real numbers only

52. Let G be an abelian group under multiplication and let $H = \{x \in G : x^2 = e, \text{ the identity}\}$
 $K = \{x^2 : x \in G\}$

Which one of the following is correct?

- (a) H is a subgroup, but K is not a subgroup
- (b) K is a subgroup, but H is not a subgroup
- (c) Both H and K are subgroups
- (d) Neither H nor K is a subgroup

53. If $f(x) = \frac{k \sin x + 2 \cos x}{\sin x + \cos x}$ is monotonically increasing, then which one of the following is correct?

- (a) $k > 2$
- (b) $k \geq 2$
- (c) $k < 1$
- (d) $k < -1$

54. If $f(x) \geq 0$ and $f'(x) \geq 0$ on an interval I, then f^2

- (a) is non-increasing on I
- (b) is non-decreasing on I
- (c) is not monotonic on I
- (d) None of the above

f > 0, f' > 0

55. Let $f(x)$ and $g(x)$ be two functions such that $g(x) = f(x) + Ax$ where $x \in [a, b]$ and A is a constant. If $g(x)$ satisfies the conditions of Rolle's theorem, then which one of the following is correct?

- (a) $A = \frac{f(b) - f(a)}{b - a}$
- (b) $A = \frac{f(b) - f(a)}{a - b}$
- (c) $A = \frac{f(b - a)}{b - a}$
- (d) $A = \frac{f(b) + f(a)}{b + a}$

g(x) = f(x) + Ax
A = 0

56. What is the value of the derivative of $|x - 2| + |x - 5|$ at $x = 3$?

- (a) -2
- (b) 2
- (c) 3
- (d) 0

2x-5
x-2 + x-5
= 2x-7

57. If one root of the equation $(a - b)x^2 + ax + 1 = 0$ is double that of the other, then what is the greatest value of b ?

- (a) 9/8
- (b) 8/9
- (c) -9/8
- (d) -8/9

58. Consider the following statements :

1. There exist 3×3 matrices A, B over reals such that AB is invertible but either A or B is not invertible.
2. Let A be a square matrix over reals. If the adjoint of A is invertible, then A is invertible.

Which of the statements given above is/are correct?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

59. If z is a complex number, then what is the locus of $\text{amp} \left(\frac{z-1}{z+1} \right) = \frac{\pi}{4}$?

- (a) Straight line
- (b) Ellipse
- (c) Hyperbola
- (d) Circle

Circle

60. If $P = \{x \in \mathbb{R} : f(x) = 0\}$ and $Q = \{x \in \mathbb{R} : g(x) = 0\}$, then what is $P \cap Q$ equal to?

- (a) $\{x \in \mathbb{R} : f(x) + g(x) = 0\}$
- (b) $\{x \in \mathbb{R} : f(x) - g(x) = 0\}$
- (c) $\{x \in \mathbb{R} : f(x) \cdot g(x) = 0\}$
- (d) $\{x \in \mathbb{R} : [f(x)]^2 + [g(x)]^2 = 0\}$

61. The vertex of the parabola is at (2, 1) and the directrix is $x - y + 1 = 0$. Where is the focus of the parabola?

- (a) (3, 1)
 (b) (3, 0)
 (c) (2, 3)
 (d) (4, 1)

$y = x + 1$
 $x - y + 1 = 0$
 (2, 1)

62. If $f(a) = (\ln a)^{-1}$, $a > 1$ and $f(x) = \int a^x dx + k$ (where k is a constant of integration), then what is $f(x)$ equal to?

- (a) $\frac{a^x - a^a + 1}{\ln a}$
 (b) $\frac{a^x - a^a}{\ln a}$
 (c) $\frac{a^x + a^a + 1}{\ln a}$
 (d) $\frac{a^x + a^a - 1}{\ln a}$

$\frac{a^x - a^a + 1}{\ln a}$
 $\frac{a^x - a^a}{\ln a}$
 $\frac{a^x + a^a + 1}{\ln a}$
 $\frac{a^x + a^a - 1}{\ln a}$

63. Let $f(x) = x - |x - x^2|$, $x \in [-1, 1]$. What is the number of points at which $f(x)$ is discontinuous?

- (a) No point
 (b) 1
 (c) 2
 (d) 3

64. If $f(x)$ satisfies the requirements of Rolle's theorem in $[1, 2]$ and $f'(x)$ is continuous in

$[1, 2]$, then what is the value of $\int_1^2 f'(x) dx$?

- (a) 0
 (b) 1
 (c) -1
 (d) 3

$f(1) = 1 - |1 - 1| = 1$
 $f(2) = 2 - |2 - 4| = 0$
 $\int_1^2 f'(x) dx = f(2) - f(1) = 0 - 1 = -1$

65. What is the greatest value of $f(x) = \int_{-\frac{1}{2}}^x |t| dt$ on the interval $[-\frac{1}{2}, \frac{1}{2}]$?

- (a) $\frac{3}{8}$
 (b) $\frac{1}{4}$
 (c) $-\frac{3}{8}$
 (d) $-\frac{1}{2}$

$f(x) = \int_{-\frac{1}{2}}^x |t| dt$
 $f(x) = \frac{1}{2}x^2$
 at $x = \frac{1}{2}$, $f(x) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$
 at $x = -\frac{1}{2}$, $f(x) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$
 at $x = 0$, $f(x) = 0$

66. Let $(1x_2x_3...x_k)_2$ be the binary number obtained on converting the decimal number 10^4 .

Consider the following statements:

- $k = 13$.
- The number of $j \in \{2, 3, 4, \dots, k\}$ such that $x_j = 0$ is 8.

Which of the statements given above is/are correct?

- (a) 1 only
 (b) 2 only
 (c) Both 1 and 2
 (d) Neither 1 nor 2

67. Let n, m be positive integers. Let $x_{t-1}...x_1x_0$ be the representation of n in the binary system and let $y_{k-1}...y_1y_0$ be the representation of m in the binary system.

Let $A = \{j \in \{0, 1, 2, \dots, t-1\} : x_j = 1\}$ and $B = \{i \in \{0, 1, 2, \dots, k-1\} : y_i = 1\}$.

Consider the following statements:

- If $n > m$, then the number of elements of A will be greater than the number of elements of B .
- If $n = 1000$, then A contains exactly 6 elements and if $m = 100$, then B contains exactly 3 elements.

Which of the statements given above is/are correct?

- (a) 1 only
 (b) 2 only
 (c) Both 1 and 2
 (d) Neither 1 nor 2

68. Consider the following rings under usual addition and multiplication as ring operations :

1. $\{p + q\sqrt{2} : p, q \in \mathbb{Z}\}$

2. $\{p + q\sqrt{2} : p, q \in \mathbb{Q}\}$

Which of the above rings is/are fields ?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

69. Let $(G, *)$ be a finite group with $o(G) = n \geq 2$. For any $g \in G$, let $o(g)$ denote the smallest positive integer m such that $g^m = e$ where $g^m = g * g * g \dots m \text{ factors}$.

Let $\max_{g \in G} o(g) = 2$

Which one of the following is correct ?

- (a) $g_1 * g_2 = g_2 * g_1$ for all $g_1, g_2 \in G$ only if n is a prime number.
- (b) $(G, *)$ is abelian only if n is a composite number.
- (c) $(G, *)$ is abelian only if n is not divisible by p^2 for any prime number p .
- (d) $(G, *)$ is abelian irrespective of nature of n .

70. Consider the following statements :

- 1. Let \mathbb{R} be a ring such that $xy = 0$ in \mathbb{R} implies either $x = 0$ or $y = 0$. If r is a non-zero element of \mathbb{R} such that $r^2 = r$, then $rx = xr = x$ for any $x \in \mathbb{R}$.
- 2. In \mathbb{Z}_9 , the ring of integers with addition and multiplicative modulo 9, there exists no element $r, r \in \mathbb{R} - \{0, 1\}$ such that $r^2 = r$.

Which of the statements given above is/are correct ?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

71. Consider the following statements :

1. Let $B = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ and

$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

Then $(I - B)(I + B + B^2) = I$.

2. If T is a ring with identity 1 and if $t \in T$ is such that $t^5 = 0$ but $t^4 \neq 0$, then $(1 - t)(1 + t + t^2 + t^3) = 1$.

Which of the statements given above is/are correct ?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

72. What is $\lim_{x \rightarrow \infty} (\sqrt[3]{1-x^3} + x)$ equal to ?

- (a) 0
- (b) 1
- (c) -1
- (d) It does not exist

Handwritten solution for Q72:

$$\sqrt[3]{1-x^3} + x = \frac{1-x^3 + x^3 + x^3}{\sqrt[3]{1-x^3} + x} = \frac{1+x^3}{\sqrt[3]{1-x^3} + x}$$

$$\lim_{x \rightarrow \infty} \frac{1+x^3}{\sqrt[3]{1-x^3} + x} = \lim_{x \rightarrow \infty} \frac{x^3}{x^3} = 1$$

73. If $[.]$ denotes the greatest integer function, then what is $\int_a^b [x] dx + \int_a^b [-x] dx$ equal to ?

- (a) $a - b$
 (b) $2(a - b)$
 (c) 0
 (d) $b - a$

$b - a +$

$f(x) = \dots$
 $f(x) = \dots$
 $f(x) = \dots$

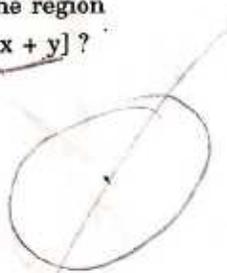
74. What is the particular integral of

$$\frac{d^2y}{dx^2} - (a+b) \frac{dy}{dx} + aby = \varphi(x)?$$

- (a) $e^{ax} \int [e^{(a-b)x} (\int \varphi e^{bx} dx)] dx$
 (b) $e^{ax} \int [e^{-(a-b)x} (\int \varphi e^{-bx} dx)] dx$
 (c) $e^{-ax} \int [e^{-(a-b)x} (\int \varphi e^{bx} dx)] dx$
 (d) None of the above

75. What is the area of the region $[(x, y) : x^2 + y^2 \leq 1 \leq x + y]$?

- (a) $(\pi - 1)/2$
 (b) $(\pi - 2)/4$
 (c) $(\pi + 1)/2$
 (d) $(\pi + 2)/4$



76. Let the unit vectors \vec{a} and \vec{b} be inclined at an angle 2θ ($0 \leq \theta \leq \pi$) and $|\vec{a} - \vec{b}| < 1$. Which one of the following is correct ?

- (a) $[0, \frac{\pi}{6}] \cup [\frac{5\pi}{6}, \pi]$
 (b) $[\frac{\pi}{6}, \frac{\pi}{2}] \cup [\frac{5\pi}{6}, \pi]$
 (c) $[0, \frac{\pi}{6}] \cup [\frac{\pi}{2}, \frac{5\pi}{6}]$
 (d) None of the above

77. What is the equation of a right circular cylinder of radius 2 and having its axis along the z-axis ?

- (a) $x^2 + y^2 = 4z^2$
 (b) $x^2 + y^2 = 2z^2$
 (c) $x^2 + y^2 = 4$
 (d) $x^2 + y^2 = 2$

78. Which one of the following system of curves is the orthogonal trajectories of the curves $r\theta = a$?

- (a) $r^2 = ce^{\theta^2}$
 (b) $r^2 = ce^{-\theta^2}$
 (c) $r^2 = ce^{2\theta}$
 (d) $r^2 = ce^{-2\theta}$

79. What is the differential equation of parabolas having their foci at the origin and axes along the x-axis ?

- (a) $y = (2x + yy') y'$
 (b) $y = (2x - yy') y'$
 (c) $y = (-2x + yy') y'$
 (d) $y = -(2x + yy') y'$

$y = \dots$

80. If $x^m y^n$ is an integrating factor of $(y^2 + 2x^2y) dx + (2x^3 - xy) dy = 0$, then what are the values of m and n respectively?

- (a) $-1/2, -1/2$
- (b) $-1/2, -5/2$
- (c) $-5/2, -5/2$
- (d) $-5/2, -1/2$

81. Let α, β, γ be the roots of the equation $x^3 - 8x + 6 = 0$.

Consider the following statements :

1. $\alpha^2 + \beta^2 + \gamma^2 = p^4$ for some prime number p .
2. $\alpha^2 \beta^2 + \beta^2 \gamma^2 + \gamma^2 \alpha^2 = n^3$ for some composite number n .

Which of the statements given above is/are correct?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

82. If α and β are different complex numbers with $|\beta| = 1$, then what is the value of $\left| \frac{\beta - \alpha}{1 - \bar{\alpha}\beta} \right|$?

- (a) 0
- (b) $1/2$
- (c) 1
- (d) 2

83. Let C be the set of complex numbers. Let $g : C \rightarrow C$ be such that $g(1) = 1$ and $g(z^2) = (g(z))^2$ for any $z \in C$. Which one of the following is correct?

- (a) There can be only two possibilities for $g(i)$, either $g(i) = 1$ or $g(i) = -1$.
- (b) There can be only three possibilities for $g(i)$.
- (c) There can be only four possibilities for $g(i)$.
- (d) There are infinite number of possibilities for $g(i)$.

84. Let \mathbb{R} denote the field of real numbers. Consider the group $GL_2(\mathbb{R})$, the group of 2×2 invertible matrices over \mathbb{R} under matrix multiplication. Fix $A \in GL_2(\mathbb{R})$.

Consider the mapping

$$f : GL_2(\mathbb{R}) \rightarrow GL_2(\mathbb{R})$$

$$B \rightarrow ABA^{-1}$$

Which one of the following is correct?

- (a) f is one-one but not onto
- (b) f is onto but not one-one
- (c) f is one-one and onto
- (d) f is neither one-one nor onto

85. If \vec{h} is any arbitrary vector, then the general solution of the equation $\vec{x} \cdot \vec{a} = p$ is expressed as

- (a) $(\vec{h} \times \vec{a}) + p \frac{\vec{a}}{a^2}$
- (b) $(\vec{h} \times \vec{a}) - p \frac{\vec{a}}{a^2}$
- (c) $-(\vec{h} \times \vec{a}) + p \frac{\vec{a}}{a}$
- (d) $(\vec{h} \times \vec{a}) + p \frac{\vec{a}}{a}$

86. What is the general solution of

$$\left(\frac{dy}{dx}\right)^2 + x\left(\frac{dy}{dx}\right) - y = 0?$$

- (a) $cx - x^2$
- (b) $c^2x + c$
- (c) $c^2x - c$
- (d) $cx + c^2$

where c is a constant.

87. Consider the following statements :

1. It is impossible to find a polynomial equation $f(x) = 0$ with real coefficients whose only roots are i , $-i$ and $1 + 2i$.
2. If $f(x) = 0$ is a polynomial equation with real coefficients and if $f(a + ib) = 0$, then $f(\pm a \pm ib) = 0$.

Which of the statements given above is/are correct ?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

88. Consider the following statements :

1. If $n, n+1, n+2$ are the roots of $x^3 - 6x^2 + 11x - 6 = 0$ for some natural number n , then any root of $6x^3 - 11x^2 + 6x - 1 = 0$ will be a rational number.
2. There exists a cubic polynomial $f(x)$ with integer coefficients such that $f(x) = 0$ has no multiple roots but $f'(x) = 0$ has multiple roots.

Which of the statements given above is/are correct ?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

89. Consider the following statements :

1. There exists a cyclic group G admitting elements g, x ($x \neq e$) such that $g^n \neq e$ for any $n \in \mathbb{N}$ but $x^t = e$ for some positive integer t .
2. Let $G = \langle g \rangle$, where $\langle g \rangle$ represents the cyclic group generated by g with $o(G) = 18$. If $H = \langle g^2 \rangle \cap \langle g^3 \rangle$, then the number of elements in H equals 3.

Which of the statements given above is/are correct ?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

90. Consider the group $G = \mathbb{C}^*$ of non-zero complex numbers under multiplication. Let $f: \mathbb{C}^* \rightarrow \mathbb{C}^*$ be given by $f(z) = z^{16}$ for any $z \in \mathbb{C}^*$. Let $\omega \in \mathbb{C}^*$. What is the number of $z \in \mathbb{C}^*$ such that $f(z) = \omega$?

- (a) 0
- (b) 2
- (c) 8
- (d) 16

91. Consider the following statements :

1. Let $S = 2\mathbb{Z} \cup 3\mathbb{Z}$ where \mathbb{Z} denotes the ring of integers under algebraic addition and multiplication. If $p, q \in S$, then $pq \in S$. However, there exist $m, n \in S$ such that $m - n \notin S$.
2. Let \mathbb{R} be a ring of real numbers under algebraic addition and multiplication. Let $n > 1$ be such that $x^n = x$ for all $x \in \mathbb{R}$. If $a, b \in \mathbb{R}$ are such that $ab = 0$, then $ba = 0$.

Which of the statements given above is/are correct ?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

92. Consider the following statements :

- Let $(T, +, \cdot)$ be a ring such that $(T, +)$ is a cyclic group. Then $(T, +, \cdot)$ is a commutative ring.
- Let $n \geq 2$. Consider (T, \oplus, \otimes) where $T = \{0, 1, 2, \dots, n-1\}$, \oplus is addition modulo n and \otimes is multiplication modulo n . If (H, \oplus) is a subgroup of (T, \oplus) , then for any $h_1, h_2 \in H$, $h_1 \oplus h_2 \in H$.

Which of the statements given above is/are correct ?

- 1 only
- 2 only
- Both 1 and 2
- Neither 1 nor 2

93. Consider the following algorithm :

Step I : get N, M

Step II : $X \leftarrow 1$
 $Y \leftarrow 1$

Step III : For $K = 1$ to M
do $X \leftarrow (N - K + 1) * X$
 $Y \leftarrow ((N - K + 1)/K) * Y$

Step IV : output X, Y

If $N = 10$ and $M = 6$, what are the values of X and Y respectively in the above algorithm ?

- 720, 120
- 720, 240
- 151200, 210
- None of the above

94. Consider the following statements :

- Let R be a ring with identity 1 such that $xy = 0$ implies $x = 0$ or $y = 0$. If $r_1, r_2 \in R$ are such that $r_1 r_2 = 1$, then $r_2 r_1 = 1$.
- Let R be an integral domain with identity. If $r \in R$ is such that r is invertible under multiplication and $r = r^{-1}$, then there can be at most two such r .

Which of the statements given above is/are correct ?

- 1 only
- 2 only
- Both 1 and 2
- Neither 1 nor 2

95. Which one of the following is an odd permutation ?

- $(1\ 2\ 3) \cdot (4\ 5\ 6)$
- $(1\ 2) \cdot (2\ 5\ 3\ 6) \cdot (1\ 3\ 2\ 4)$
- $(6\ 4\ 3\ 2\ 1)$
- $(1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9)$

96. A certain memory has a capacity of $16\text{ K} \times 32$. How many memory cells does it contain ?

- 32
- 16384
- 524288
- None of the above

97. NOR operation for two inputs X and Y is given as

- $X + Y$
- XY
- $\overline{X} \overline{Y}$
- $\overline{X} + \overline{Y}$

98. The line $y = 1 - x$ and the curve $y = \ln x$ on $(0, \infty)$

- intersect each other at an angle $\pi/3$
- never intersect each other
- intersect each other at an angle $\pi/6$
- intersect each other at right angles

99. If f_1 and f_2 are the integrating factors of the differential equations $x \frac{dy}{dx} + 2y = 1$ and

$\frac{dy}{dx} - \frac{2}{x}y = \frac{1}{x}$ respectively, then which one of the following is correct ?

- $f_1 = x^4 f_2$
- $f_1 = x^3 f_2$
- $f_1 = x^2 f_2$
- $f_1 = f_2$

100. A plane curve has the property that the tangents from any point on the y-axis to the curve are of constant length. What are the orthogonal trajectories of the family of such curves ?

- Circles
- Ellipses
- Straight lines
- Parabolae

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